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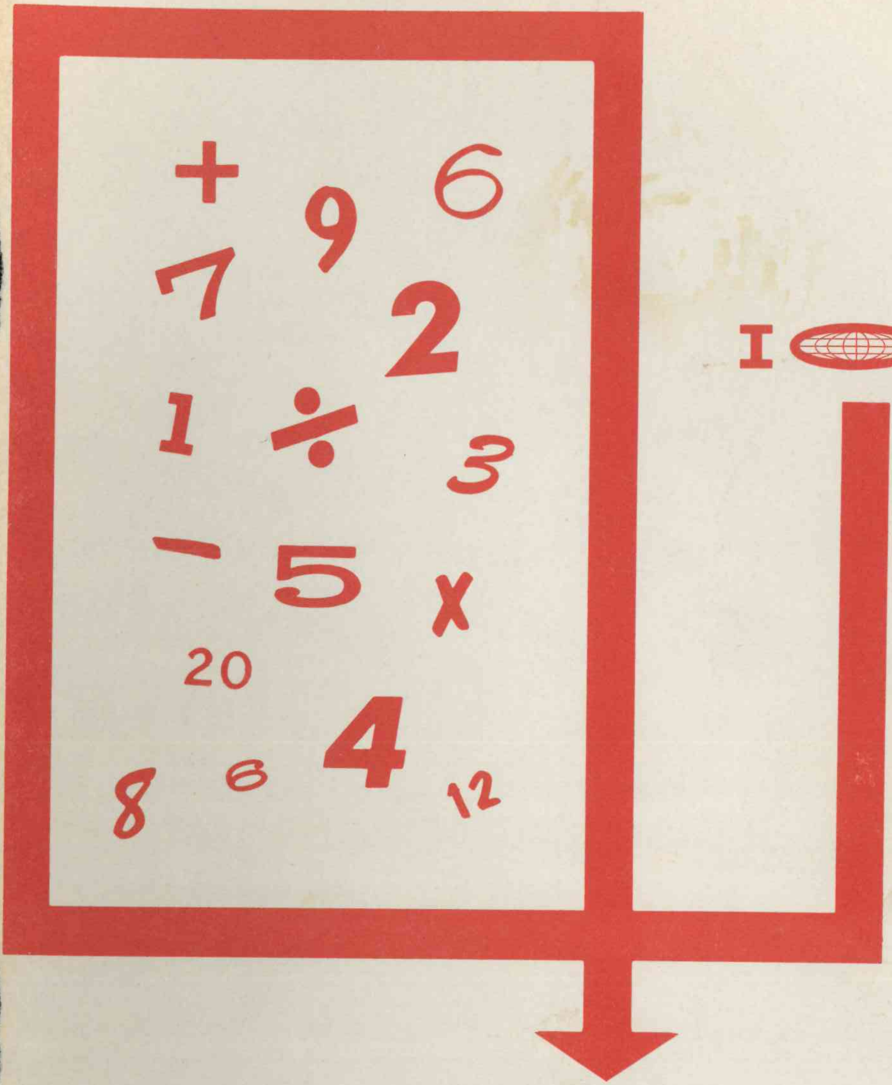

Practical Arithmetic

Weights and Measures

By

W. M. SPANGLER, M.A.

6750E
Part 5
Edition 1

Practical Arithmetic

PART 5

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What This Text Covers . . .

Almost every day you deal with various units of weight and measure in some way. In this text you are going to study the most commonly used systems of weights and measures. You must know how to use these units properly to do the work that comes later in your course. The following study outline will help you to organize your work.

1. UNITS OF WEIGHT AND MEASURE . . . . Pages 1 to 40

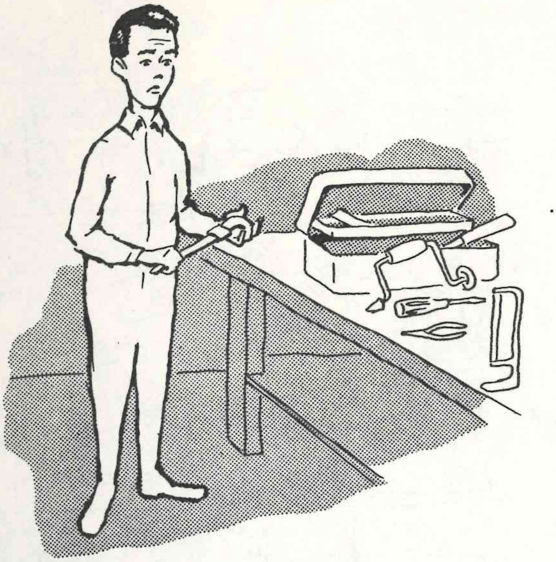
   After explaining certain terms with which you must be familiar, we discuss the measurement of lines, angles, areas, and volumes. Then we describe the avoirdupois, troy, apothecaries', and metric systems of weight. Units of time, temperature, and money are presented last. Tables of weights and measures are included.

2. CONVERSION OF UNITS . . . . Pages 41 to 52

   This section shows you how to convert from units of one kind to units of another kind.

3. CALCULATIONS WITH MEASURED QUANTITIES . . . . Pages 53 to 62

   It's really most important to be able to add, subtract, multiply, and divide denominate quantities. Furthermore, it is not difficult to perform these basic operations upon numbers involving units of weight or measure. You'll find that this is true as you proceed with your study of this section.



Practical Arithmetic

PART 5

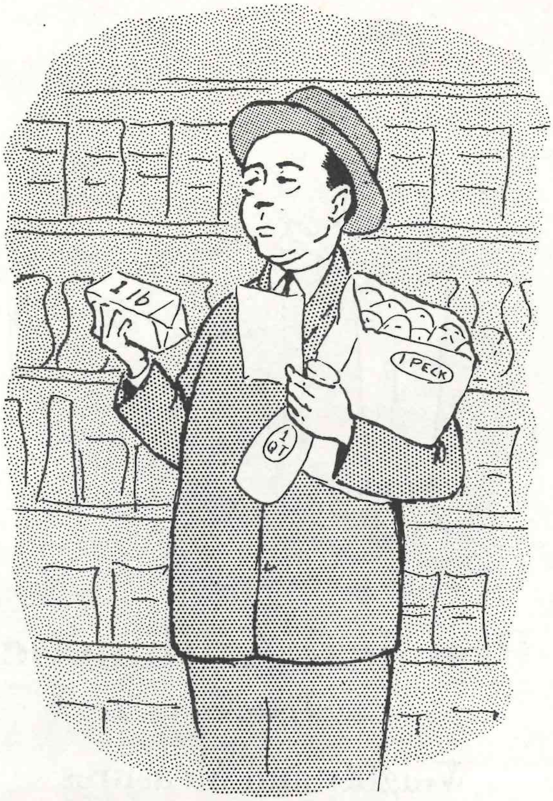
Weights and Measures

Units of Weight and Measure

Applying Your Knowledge of Arithmetic

1. Thus far in your study of *Practical Arithmetic* you have learned how to add, subtract, multiply, and divide numbers. It is true that you have learned how to do other things, such as raising numbers to powers, taking the square root of numbers, and finding numbers to bases other than 10. In all this work, however, you have dealt primarily with abstract numbers. In only a very few problems were you concerned about the quantities or things to which the numbers referred. In this text you are going to learn how to use your knowledge of arithmetic in the kind of problems that you will meet in everyday life or on your job. In many respects you are much like the person who has a fine set of tools, but who has not yet learned how to use them. Your tools are the operations that you can perform upon numbers.

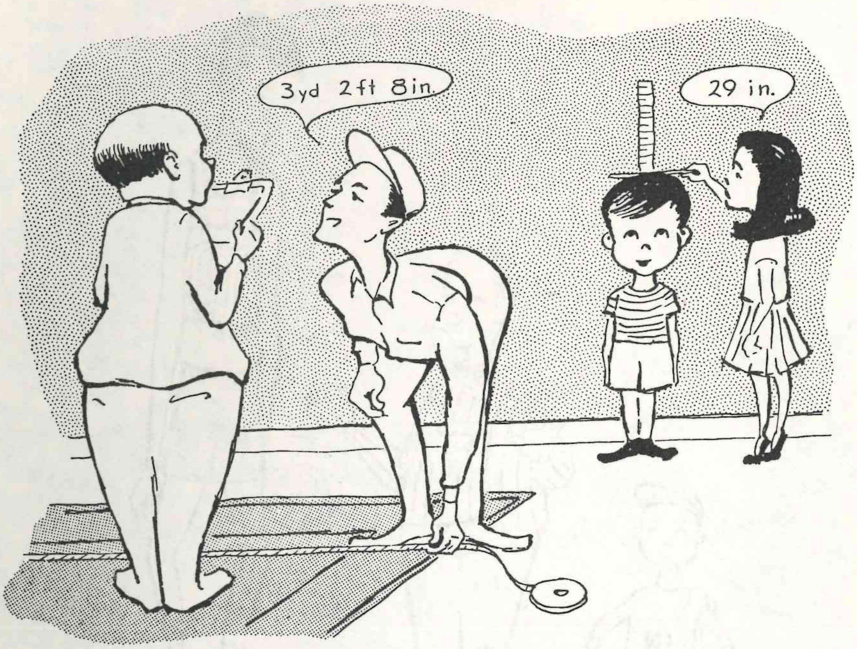
You will recognize many of the terms mentioned in this text. But don't let that lull you into a state of false security. Study *all* of this material carefully, because some of the terms you think you know may be the very things on which your knowledge is not complete.



Units of Measure

2. A unit of measure is a standard, fixed by law or by custom, that is used to measure a quantity. For example, when you buy a quart of milk, a pound of beans, or a peck of potatoes, the amount of milk, beans, or potatoes that you get is determined by how much there is in a quart, a pound, or a peck. Units of measure can be used to compare similar quantities. For example, when you know that two distances are 80 miles and 20 miles, you can easily find the difference between the distances or you can determine how much greater the first distance is than the second distance.

In the examples of the preceding paragraph the quart, the pound, the peck, and the mile are the units of measure we used. As you probably know, distances can be measured in other units, such as feet, inches, and yards. Also, there are many other units in use for measuring other quantities. For example, the gallon, the quart, and the pint are used for measuring liquids. The ton, the pound, and the ounce are used for measuring weight. The bushel, the peck, and the quart are used for measuring things like potatoes and tomatoes. The hour, the minute, and the second are used for measuring time. In this text you will learn more about


3yd 2ft 8in.
29 in.

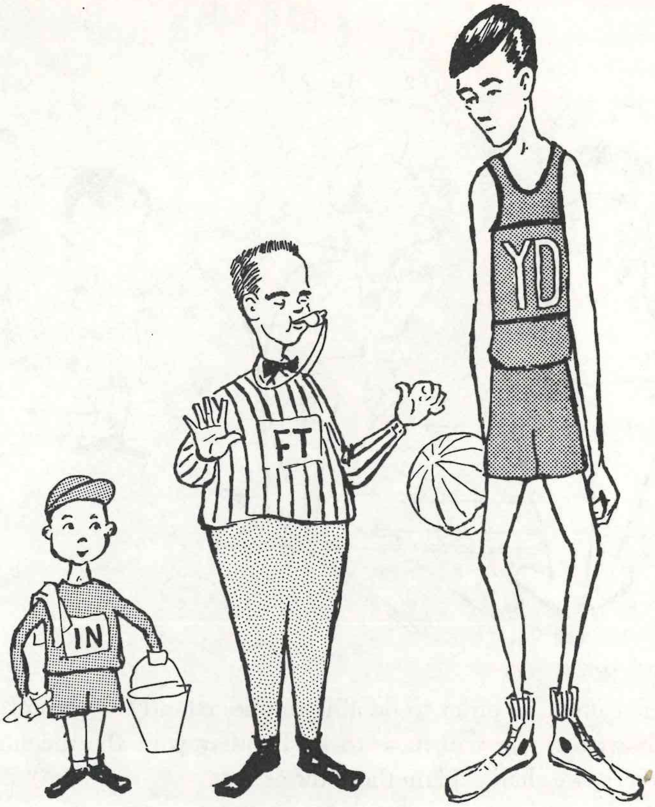
these and other units. In order to be able to understand the uses of some of these units fully, however, you will have to be familiar with the meanings of certain terms. Therefore, we shall explain these terms first.

Denominate Numbers

3. When a number is used in connection with measures, it becomes a *denominate number,* that is, a *named number.* Numbers are of the same *denomination* when they are combined with the same unit of measure, as 2 pounds and 7 pounds. A foot and an hour are also units of measure, but of different kinds. Notice that a denominate number is a special kind of *concrete number!*

If a denominate number consists of units of measure of only one denomination, it is called a *simple denominate number* or a *simple number.* For example, 14 inches is a simple number because it contains units of measure of only one denomination, namely, inches. The denominate numbers 16 cents, 10 hours, 6 gallons are all simple denominate numbers, but they are not of the same kind.

If you have a combination of simple denominate numbers of different denominations, the combination is called a *compound denominate number,* or a *compound number.* For example, 3 yards 2 feet 7 inches is a compound number because it is a combination of three different denominations: yards, feet, and inches. Notice that these denominations are of the same kind; that is, all of them are measures of length. Other examples of compound numbers are 2 pounds 3 ounces 10 grains, 5 gallons 3 quarts 1 pint, 10 hours 14 minutes 32 seconds.



A unit measure that is larger than another unit measure is said to be of a higher denomination. If it is smaller than another unit measure, it is of a lower denomination. Thus, a foot is of a higher denomination than an inch, but it is of a lower denomination than a yard.

Systems of Measure

4. In modern practice two systems of measures are employed: the English system and the metric system. The English system is in general use in the United States, Great Britain, and Canada. The metric system is used on the European continent and to some extent in the United States, as for instance, in chemistry and pharmacy. This system is a decimal system. That is, the values of the different units of the same kind increase or decrease by tens, 10 units of each denomination making 1 unit of the next higher denomination.

Abbreviations of Units

5. In writing denominate numbers, it is convenient to use abbreviations instead of writing out the name of the unit in full. For example, the names inch and inches may be abbreviated to in. Thus we write 5 in. instead of 5 inches.

Similarly, 8 ft means 8 feet. The abbreviations commonly used for the various units are given later in the text. Also given are tables which show the relation between various units of measure of the same kind. Don't try to memorize these tables as such. You will soon become familiar with the units that are most commonly used.

Check Your Learning

By now you should be familiar with this type of instruction and you should need no introduction to it.

As before, the statements are numbered consecutively throughout the text. You'll again find the correct answer slightly below and to the right of each statement. Cover our answer with a piece of paper, and complete the statement by writing the best answer you can in the space provided. Then compare your answer with ours. If your answer is correct, go on to the next statement. If your answer is not correct, review the section of the text on which the statement is based.

1. A ______________ number is a named number.

(denominate)

2. A denominate number is a special kind of ______________ number.

(concrete)

3. A simple number is a denominate number that consists of units of measure of only ________ denomination(s).

(one)

4. The combination of simple denominate numbers of different denominations is called a ______________ number.

(compound)

5. A foot is of a (higher, lower) denomination than an inch.

(higher)

6. A foot is of a (higher, lower) denomination than a yard.

(lower)

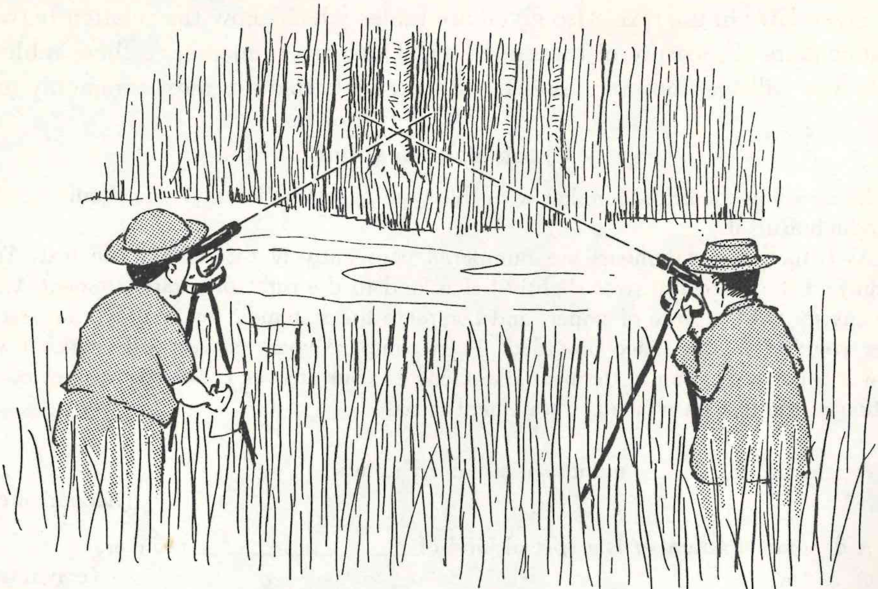
7. The abbreviation for inches is ________ .

(in.)

Lines and Points

6. You probably think of a line as a mark made with a pencil. Such a mark is one kind of line. However, a line does not have to be an actual mark. There are many other kinds of lines. For example, an edge of a sheet of paper or an edge of some other object may be considered a line.

A point may be a small dot made with a pencil. For instance, a dot like the dot in the letter i may be called a point. However, a point may also be a *position* that is not actually marked. In making a measurement on an object, you may imagine that a point is located in a certain position without actually making a



mark at the point. For instance, a corner of a sheet of paper may be considered a point.

In Fig. 1*(a)* there is a line with dots marking points at the ends of a line and at a place on the line between the ends. A similar line is shown in *(b)* without dots. However, the ends of this line are really points, and a point can be located any place on the line between its ends. In *(a)* and *(b)* we used the letters *A* and *B* to refer to the points at the ends of the line. These points are called point *A* and point *B*, and the line joining these points is called line *AB* or line *BA*. We also used the letter *C* in *(a)* to refer to a point on the line between its ends. The part of the line between *A* and *C* is called line *AC*, and the part between *B* and *C* is called line *BC*. Lines, such as *AC* and *BC*, that have definite end points are called ***line segments***. You will often see a small line written over the letters designating the end points. Thus $\overline{AC}$ and $\overline{BC}$ are ways of referring to the line segments *AC* and *BC*.

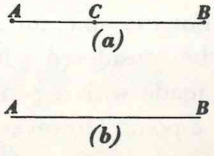
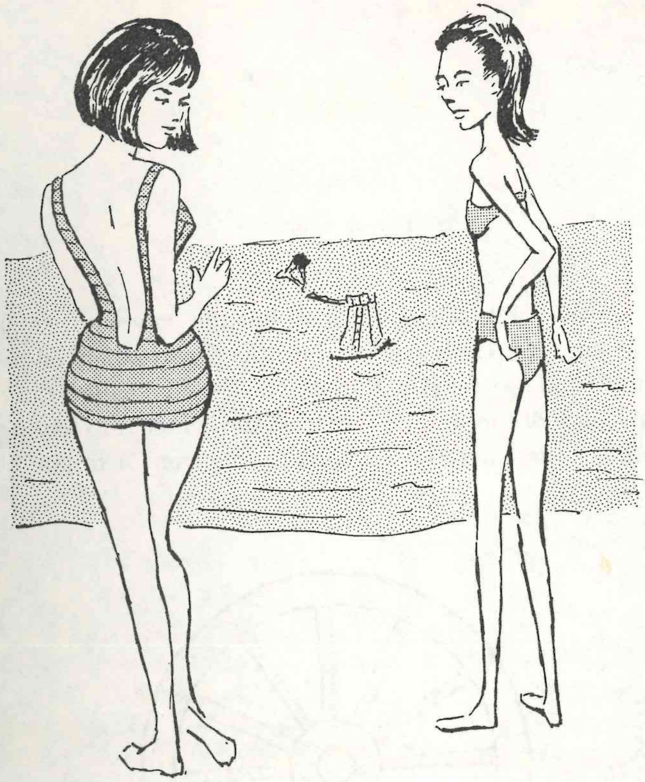


FIG. 1. STRAIGHT LINES



Straight and Curved Lines

7. Either of the lines in Fig. 1 is called a *straight line* because it is drawn directly between the two points *A* and *B*. There can be only one straight line between two given points. The length of a straight line is the distance between the end points. Thus you can find the length of the line *AB* by measuring the distance from point *A* to point *B*. Likewise, you can find the length of part *AC* by measuring the distance from *A* to *C*.

Not all lines are straight lines. Some are curved. A *curved line* is shown in Fig. 2. The length of a curved line is the distance between its ends measured along the line itself. You can see that the length of a curved line is greater than the length of the straight line that could be drawn between the ends of the curved line. As stated, the distance between two points is understood to be the length of the straight line joining the two points. If the distance along a curved line between the points is meant, the exact form of the curved line must be given.

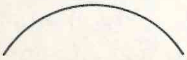


FIG. 2. CURVED LINE

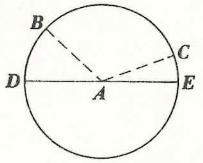
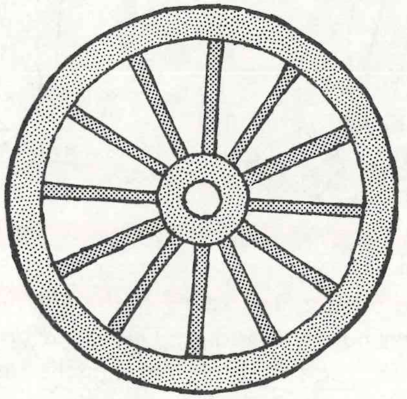


FIG. 3. CIRCLE

The Circle

8. There are many kinds of curved lines, but the only kind we shall consider in this text is the *circle*. A circle is shown in Fig. 3. All points on a circle are at the same distance from a point A within it. This point is called the *center* of the circle. Thus the length of the straight line from the center A to any point B on the circle is the same as the length of the straight line from A to any other point C on the circle.



A line from the center of a circle to any point on the circle is called a *radius* of the circle. Lines AB and AC in Fig. 3 are radii (the plural of radius) of the circle. Any line through the center of a circle joining two points on the circle is called a *diameter*. The line DE in Fig. 3 is a diameter of the circle.

The part of a circle between any two points on the circle is called an *arc* of the circle. For example, the part of the circle in Fig. 3 between the points B and C is the arc BC.

Angles

9. An *angle* is the opening between two straight lines that meet at a point. Thus, in Fig. 3, the lines AB and AC form an angle called angle BAC or angle CAB at the point A. The lines AB and AC are the *sides* of the angle, and the point A at which they meet is called the *vertex* of the angle. An angle designated

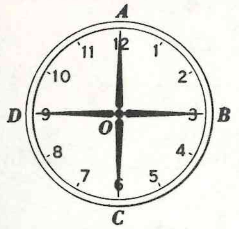


FIG. 4. POSITIONS OF MINUTE HAND OF CLOCK

by three letters always has the vertex letter in th middle, as *A* is in the middle of both *BAC* and *CAB*.

You will obtain a clearer picture of what an angle is and how angles are measured if you imagine the minute hand of a clock moving around the face of the clock. Begin when the minute hand is at 12. The radius *OA* in Fig. 4 indicates the position of the hand at this point. The radius *OB* indicates the position of the hand $\frac{1}{4}$ hour later; the radius *OC* indicates the position $\frac{1}{2}$ hour later; and the radius *OD* indicates the position $\frac{3}{4}$ hour later. And finally, $\frac{4}{4}$ of an hour later, the hand has reached its starting position *OA*.

As the hand of the clock in Fig. 4 moves from its first position, on radius *OA*, through the position on radius *OB* to the position on radius *OC*, the angle between the radius *OA* and the hand increases in size. Also, the arc of the circle between the end of the hand and its first position *A* becomes longer. You can see from this, or from an actual clock, that the angle *AOB* between the radius *OA* and the radius *OB* through which the hand passes in the first $\frac{1}{4}$ hour is equal to the angle *BOC* between the radius *OB* and the radius *OC* through which the hand passes in the second $\frac{1}{4}$ hour.

Likewise, the length of the arc *AB*, Fig. 4, through which the end of the hand moves in the first $\frac{1}{4}$ hour, is equal to the length of the arc *BC*, through which it moves in the second $\frac{1}{4}$ hour. Also, the angle *COD* and the angle *DOA*, through which the hand passes in the third $\frac{1}{4}$ hour and the fourth $\frac{1}{4}$ hour, are each equal to the angle *AOB*. For the same reason the length of the arc *CD* or the arc *DA* is equal to the length of the arc *AB*. Therefore, the angle between any two radii of a circle can be measured by the length of the arc of the circle between the radii. Thus the angle *AOB*, Fig. 4, may be measured by the length of the arc *AB*, and the angle *BOC* may be measured by the length of the arc *BC*.

Check Your Learning

8. A point may be a ______________ that is not actually marked.

(position)

9. Lines that have definite end points are called _______ __________.

(line segments)

10. There can be only ________ straight line between two given points.
(one)

11. A straight line is the ________________ distance between two points.
(shortest)

12. All points on a circle are at the ________ distance from the center of the circle.
(same)

13. A line from the center of a circle to any point on the circle is called a(n) ____________.
(radius)

14. The part of a circle between any two points on the circle is called a(n) ____________ of the circle.
(arc)

15. A straight line that passes through the center of a circle and joins two points on the circle is a(n) ____________.
(diameter)

16. The length of an arc of a circle measures the ____________ formed by the radii.
(angle)

Units of Measure of Angles

10. For the purpose of measuring angles, a circle is divided into 360 equal parts. Each such part is called a *degree*. Thus there are 360 degrees in every circle or complete revolution. A degree is further divided into 60 equal parts each of which is called a *minute*. Each minute is divided into 60 *seconds*, as shown in Table 1.

As you may already know, the word "degree" is also the name of a unit for measuring temperature. And the minute and the second are also the names of units of time. So, unless there is no doubt that the words "degree," "minute," and "second" refer to angles, you should use the terms "degrees of angle," "minutes of angle," and "seconds of angle."

Degrees, minutes, and seconds of angle are usually indicated by the symbols °, ′, and ″. Thus, 25°18′30″ means 25 degrees 18 minutes 30 seconds. You will note that the symbols for minutes and seconds of angle are the same as the symbols for feet and inches. However, you will always know which units are meant when these symbols are used.

TABLE 1

COMMON UNITS FOR MEASURING ANGLES

1 degree = 60 minutes
1 minute = 60 seconds
1 degree = 3600 seconds

When the size of an angle or an arc is not a whole number of degrees, a part of a degree may be indicated by a decimal or a fraction. Thus, an angle may be given as 36.25° or as 36¼°. However, since a degree is divided into 60 minutes and a minute is further divided into 60 seconds, part of a degree can also be expressed in minutes or in minutes and seconds. Thus 36.25° can be expressed as 36°15′00″.

Right Angle

11. An angle that contains 90° is commonly called a *right angle.* Since there are 360° in an entire circle, a right angle is one-fourth of a circle. Thus each of the four angles *AOB, BOC, COD,* and *DOA* in Fig. 4 is a right angle.

The two lines that form a right angle are said to be perpendicular, perpendicular to each other, or at right angles to each other. Thus the straight lines *AC* and *BD* in Fig. 4 are perpendicular.

Practice Problems

First try to find the answer to each of the following problems without referring to the preceding explanations. If you do not know how to find the answer to a problem, try to find how to do it in the preceding parts of this text before you look at the answer given at the back of the text. After you have studied the next portion of the text, come back to these problems and see if you can answer them without referring to the text.

1. How many degrees are there in 540′? 9°
2. Express 23.75° in degrees and minutes. 23°45′
3. Use symbols to write two degrees thirty-five minutes fourteen seconds.
4. How many minutes are there in 45°? 2700
5. Express 36.5° in degrees and minutes. 36°30′
6. Write sixteen degrees twenty-two minutes five seconds by using symbols.
7. How many seconds are there in 41′? 2460

Units of Length in the English System

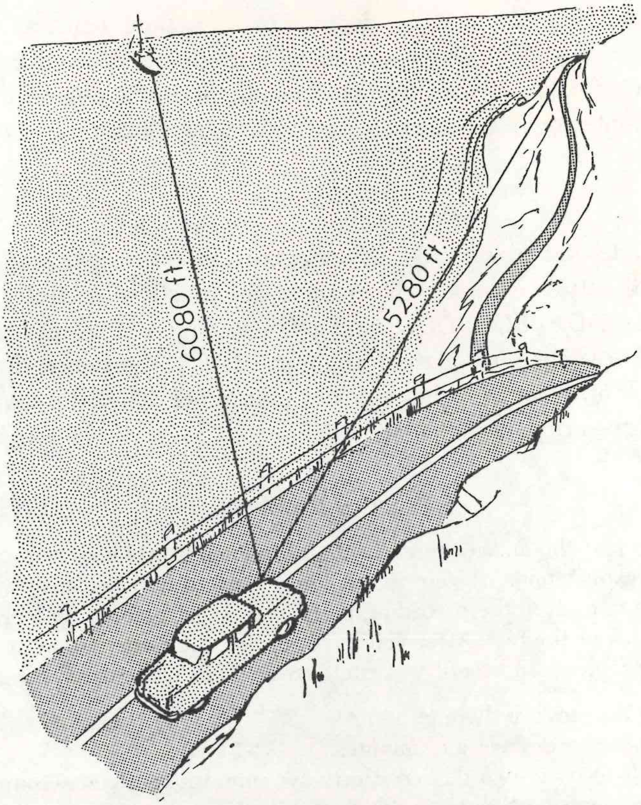
12. Any unit of measure used to give the length of a line is called a unit of length. Thus, the mile, the yard, the foot, and the inch are units of length. These units are used in a system of measures known as the *English system.* As we said before, this system is used for ordinary purposes in the United States, Canada, and Great Britain.

The relations between the most commonly used units of length in the English

TABLE 2

UNITS OF LENGTH IN ENGLISH SYSTEM

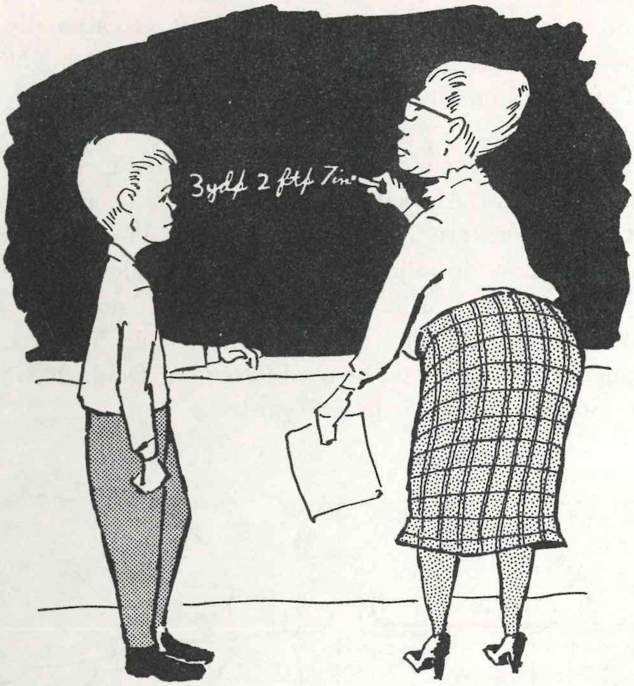
1 foot	=	12 inches	1 rod	=	16½ feet
1 yard	=	3 feet	1 furlong	=	220 yards
1 mile	=	5280 feet	1 fathom	=	6 feet
			1 nautical mile	=	6080 feet


6080 ft.
5280 ft.

system are given in the left-hand portion of Table 2. In the right-hand part of the table there are units of length that are used less often. It is common practice, when the word "mile" is used without any special explanation, that a *statute mile* of 5280 feet is understood. As a matter of fact, the term "statute mile" is seldom used in ordinary speech and writing. This kind of mile is the unit for measuring distances on land, on a lake, or on a river. For measuring distances on an ocean, the nautical mile – 6080 feet – is used. Unless it is obvious that a nautical mile is meant, the word "nautical" should never be omitted when referring to this unit.

The *rod* is used mostly for measuring the length of a fence. There are 320 rods in a mile. The *furlong* is used for measuring distances in horse racing and sometimes in foot racing. There are 8 furlongs in a mile. The *fathom* is used for measuring the depth of the water in an ocean or other body of deep water.

The abbreviations for English units of length are in. for inch or inches, ft for foot or feet, and yd for yard or yards. A period is used only in the abbreviation for inch. It is used there so that the abbreviation and the word "in" will not be confused. A period is not needed, and it is customary not to use it, in the abbreviations for foot and yard. Notice that each abbreviation stands for both the


3yd~~s~~ 2 ft~~s~~ 7in.

singular and plural name of the unit. Never add an s to any of these abbreviations to make it plural.

As we have already mentioned, on drawings and in some calculations the symbol ′ is commonly used to indicate feet and the symbol ″ to indicate inches. Thus $2'6\frac{1}{2}''$ means 2 feet $6\frac{1}{2}$ inches. From this point on in the text, we shall use abbreviations for most units of measure after we have introduced them. This will help you become familiar with the abbreviations.

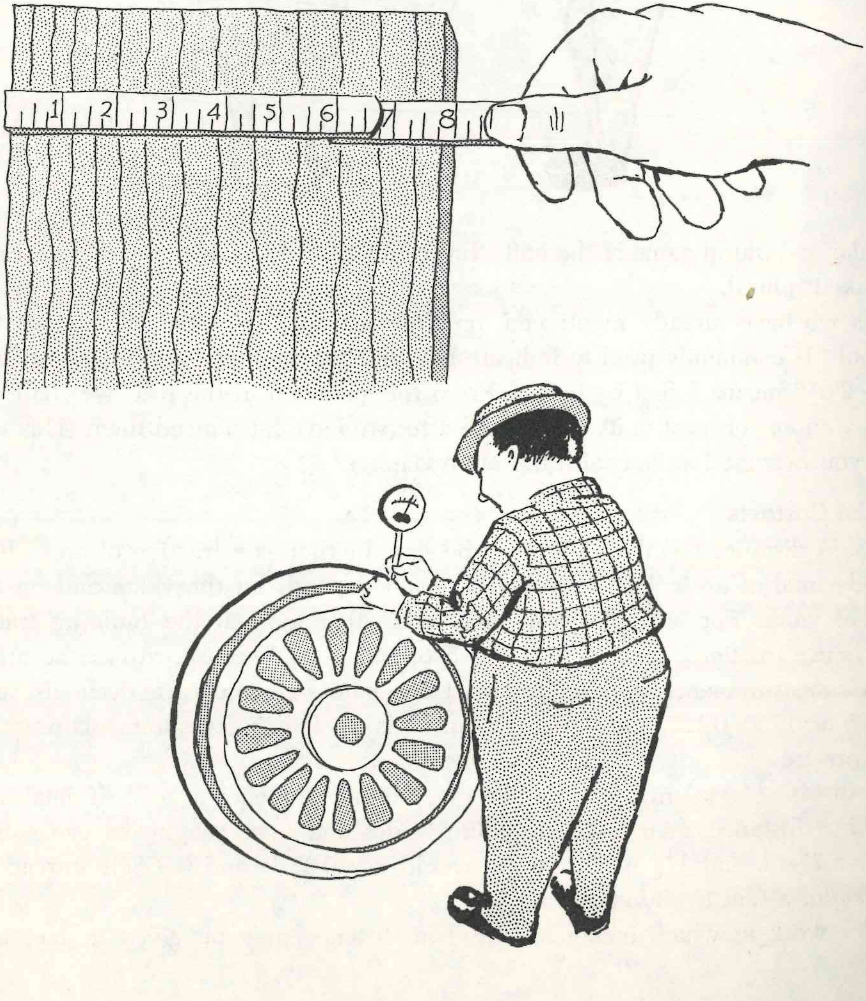
Special Customs

13. A distance less than 1 in. may be stated either as a fraction of an inch or as a decimal of an inch. The way it is given depends on the value and on the kind of work. For example, carpenters and other men in the building trades always use fractions, such as 1/2, 3/4, 5/8, and 9/16, because extreme accuracy in their measurements is seldom required. Mechanics generally use decimals, such as 0.25 or 0.250, 0.125, and 0.0625, because many of their measurements must be very precise.

A distance less than 1 ft is usually given either in inches or as a decimal of a foot. For instance, men in the building trades and many engineers use values such as $7\frac{3}{4}$ in. and $11\frac{1}{2}$ in. Surveyors would use 0.65 ft and 0.958 ft instead. A fraction of a foot is seldom used.

For work in which inches are used, a distance may be given in feet and inches, as for example, 4 ft $8\frac{1}{2}$ in. Where inches are not used, a distance may be given in feet and a decimal part of a foot, as 18.15 ft. Occasionally a number of inches is changed to a fraction of a foot. For instance, $18\frac{1}{2}$ ft might very well be written instead of 18 ft 6 in. However, you would never write $18\frac{7}{12}$ ft instead of 18 ft 7 in.

A distance that is not a whole number of miles may be given either as a mixed number (an integer with a fractional part) or as a mixed decimal (an integer with a decimal part). Examples are $10\frac{3}{4}$ miles and 18.2 miles. It is not a common practice to give a distance in miles and yards or in miles and feet. If a value expressed with a simple fraction of a mile or expressed to the nearest tenth of a mile is not accurate enough, the entire distance should be given in feet or in yards. For instance, 8240 feet would be used instead of 1 mile 2960 feet, and 3000 yards would be used instead of 1 mile 1240 yards.



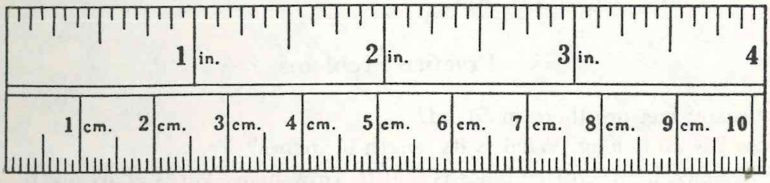
Units of Length in the Metric System

14. The basic unit of length in the metric system is the *meter.* Larger units are defined by multiplying the meter by 10 or by a multiple of 10. The only larger unit in common use is the *kilometer,* which is 1000 meters.

Units smaller than the meter are defined by dividing the meter by 10 or by a multiple of 10. Only two of the smaller units are in common use. These are the *centimeter* and the *millimeter.* There are 100 centimeters in 1 meter. Therefore, 1 centimeter is $\frac{1}{100}$, or 0.01, meter. There are 1000 millimeters in 1 meter, so 1 millimeter is $\frac{1}{1000}$, or 0.001, meter. And, of course, if 1 centimeter is 10 millimeters, then 1 millimeter is $\frac{1}{10}$ centimeter.

The abbreviations for the common metric units of length are m for meter or meters, km for kilometer or kilometers, cm for centimeter or centimeters, and mm for millimeter or millimeters. No period or s is used with any of these abbreviations.

A meter is equal to 39.37 in. But a meter is also equal to 100 cm, so 39.37 in. equals 100 cm. The number of centimeters in an inch is the quotient obtained by dividing 100 by 39.37, or $100 \div 39.37 = 2.54$. Thus, 1 in. is equivalent to 2.54 cm.



Check Your Learning

17. The mile, the yard, the foot, and the inch are units of measure in the ____________ system of measure.

(English)

18. There are 5280 ft in a ____________ .

(mile)

19. The abbreviation for foot and feet is ________ .

(ft)

20. The abbreviation for meter or meters is ________ .

(m)

21. The abbreviation mm is used for ____________ .

(millimeter)

22. The basic unit of length in the metric system is the ____________ .

(meter)

23. Units smaller than a meter are defined by dividing the meter by ten or by a multiple of ________ .

(10)

24. There are ________ feet in one yard.

(3)

25. There are 12 in. in ________ ft.

(one)

26. A meter contains ________ cm.

(100)

27. There are ____________ in. in a meter.

(39.37)

EXAMPLE 1. A piece of shafting is 7 ft long. What is its length in inches?

SOLUTION. According to Table 2, 1 ft = 12 in. Therefore, 7 ft must be equal to 7 × 12 = 84 in. Ans.

EXAMPLE 2. The width of a door is 48 in. What is the width in feet?

SOLUTION. According to Table 2, 12 in. = 1 ft. Therefore, the number of feet in 48 in. is 48 ÷ 12 = 4 ft. Ans.

EXAMPLE 3. How many feet are there in 3 statute miles?

SOLUTION. According to Table 2, 5280 ft = 1 mile. Therefore, 3 miles must be equal to 3 × 5280 = 15,840 ft. Ans.

Practice Problems

1. How many feet are there in 51 yd?
2. A board is 10 ft long. What is its length in inches?
3. The distance between two poles is 300 ft. How many yards apart are they?
4. How many feet are there in 10 miles?
5. How many centimeters are there in 6 meters?
6. A bolt is 3 in. long. What is its length in centimeters?
7. A ship traveled 36,480 ft. How many nautical miles did it travel?
8. A field is 16 rods long by 30 rods wide. What are these measurements in feet?
9. How many inches are there in 25 meters?
10. A wire is 630 cm long. Give the length in meters.

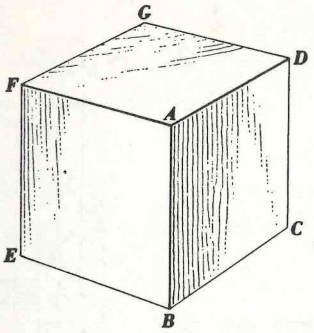


FIG. 5. SURFACES

Measurement of Areas and Volumes

Surfaces

15. The top of a table is a surface, and the outside of a ball is also a surface. The table top and many other surfaces, such as a wall of a room, a blackboard, or a pane of glass, are intended to be flat. Such surfaces are called *plane surfaces* or *planes*. Other surfaces, such as the outside of a ball, a pipe, or a funnel, are *curved surfaces*. An object shaped like a pail has both a flat, or plane, surface and a curved surface.

Any plane surface of an object is bounded by definite lines. Each distinct part of the outside of an object is usually considered a separate surface. For example, when you look at a block of wood shaped like that shown in Fig. 5, you see three separate surfaces. One of these surfaces is bounded by the four straight lines *AB*, *BC*, *CD*, and *DA*. Another surface is bounded by the four straight lines *AB*, *BE*, *EF*, and *FA*. The third surface is bounded by the four straight lines *AD*, *DG*, *GF*, and *FA*. The block also has three other surfaces. These cannot be seen when you look at the three surfaces shown, but they are similar to the visible surfaces.

Square

16. In order to understand what is meant by the area of a surface and to learn how units of area are used, you must know what a *square* is. A square is represented in Fig. 6. It is a plane surface, or part of a plane surface, bounded by four straight lines. All four of these lines must have the same length, and they must so meet one another that they form a right angle at each corner of the square. In other words the sides are perpendicular, or they are "square." Thus in Fig. 6

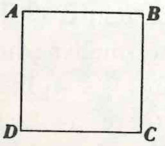


FIG. 6. SQUARE

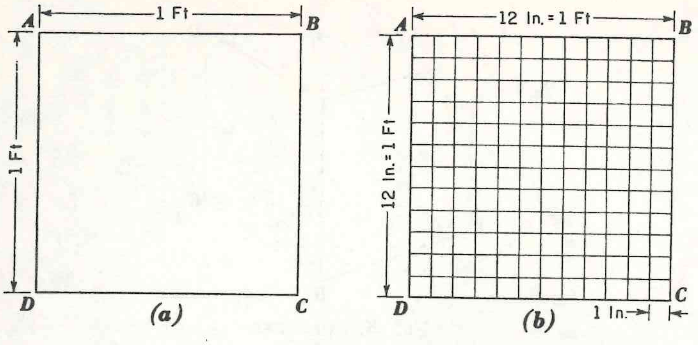


FIG. 7. RELATION BETWEEN UNITS OF AREA

the four lines *AB*, *BC*, *CD*, and *DA* have the same length and each of the angles is a right angle. Each of the four lines is called a side of the square.

Units of Area

17. The size of a surface or of a part of a surface is its *area*. The term "area" means the extent of a surface within its boundary lines. For example, the area of a floor is the extent of the visible surface limited by the four surrounding walls. Since area refers to extent of surface, its unit of measurement must be a surface also. The unit of area is a square.

Various units are used to measure areas. A unit of area is the size of a surface included in a square each side of which is of unit length. If the length of each side of the unit square is 1 in., the unit of area is a *square inch*. If the length of each side of the unit square is 1 ft, the unit of area is a *square foot*. If the length of each side of the unit square is 1 cm, the unit of area is a *square centimeter*. Thus the name of each such unit of area consists of the word "square" in front of the name of a unit of length. The abbreviation for any unit of area of this kind is the abbreviation sq for the word "square" plus the abbreviation of the name of the unit of length. Thus, sq in. stands for square inch and sq ft stands for square foot.

How Units of Area Are Related

18. To see the relation between a unit of area with one name and a unit of area with another name, compare *(a)* and *(b)* of Fig. 7. In *(a)* each side of the square *ABCD* is 1 ft long and the area of the square is 1 sq ft. Also, the length of each of the four lines *AB*, *BC*, *CD*, and *DA* in *(b)* is 1 ft, and the square *ABCD* has an area of 1 sq ft. This area of 1 sq ft is shown separated into a number of smaller squares. Each side of these smaller squares has a length of 1 in. So the area of each of these smaller squares is 1 sq in.

Since there are 12 in. in 1 ft, there are 12 small squares in each row across Fig. 7*(b)* and there are 12 rows. Therefore, the total number of small squares in

TABLE 3

UNITS OF AREA

1 square foot	=	144 square inches
1 square yard	=	9 square feet
1 acre	=	43,560 square feet
1 square mile	=	640 acres
1 square centimeter	=	100 square millimeters

the square *ABCD* is 12 × 12, or 12^2. The number 12^2 is equal to 144. In other words, 1 sq ft contains 144 sq in., or the number of square inches in a square foot is the square of the number of inches in a foot.

The observation of the preceding paragraph should suggest a general procedure, namely, that no matter which two units of area you are dealing with, you can find how many of the smaller units are contained in the larger unit by squaring the number of smaller linear units that are contained in the larger linear unit. For example,, since there are 3 ft in 1 yd, the number of square feet in a square yard is 3^2, or 9. In Part 4 of *Practical Arithmetic* we used a similar method to explain why the second power of a number is called its *square*.

A special unit of area commonly used for measuring land is the *acre*. There are 640 acres in a square mile. Since the number of square feet in a square mile is 5280^2, or 27,878,400, you can easily see that the number of square feet in an acre is 27,878,400 ÷ 640, or 43,560. The relations between the common units of area are shown in Table 3.

Area of a Square

19. We shall now explain the procedure for finding the area of a square when you know the length of a side of the square. Applying this procedure will fix the ideas concerning units of area in your mind. All you have to do is to find the square, or second power, of one side. However, you must be careful to use the proper units.

EXAMPLE 1. If each side of a square is 6 ft long, what is the area of the square in square feet?

SOLUTION. To find the area in square feet, square the length of a side in feet. The product of 6 and 6 is 36. So

$$\text{Area} = 6^2 = 6 \times 6 = 36 \text{ sq ft} \quad \text{Ans.}$$

EXAMPLE 2. Each side of a square is 4.6 in. long. Find the area of the square in square inches to the nearest tenth of a square inch.

SOLUTION. First you should consider the units in which the area is to be expressed. Since you want the area in square inches and the length of the side is given in inches, you need only square the length of the side. The work is as follows:

$$\text{Area} = 4.6^2 = 4.6 \times 4.6 = 21.16, \text{ or } 21.2 \text{ sq in.} \quad \text{Ans.}$$

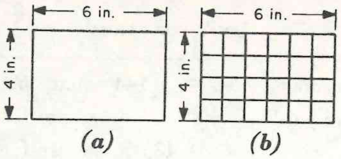


FIG. 8. RECTANGLE

The Difference Between Square Feet and Feet Square

20. Expressions such as 4 square feet and 4 feet square do not mean the same thing. When we say a surface is equal to 4 sq ft, we refer to its area. And from the preceding paragraphs we know that, if a square is said to have an area of 4 sq ft, each of its sides must have a length of 2 ft, because 2 × 2 = 4 sq ft.

The expression "4 feet square" does *not* refer to the area of a surface. It does refer to the length of the sides of a square, and it means that each side of the square is 4 ft in length. The area of this square, therefore, is 4 × 4 = 16 sq ft.

Application of Square Measure

21. A square inch, a square foot, a square yard, and so on, belong to the units of *square measure,* which is employed for measuring the extent of areas such as floors, building lots, and farms. Square measure is also used to determine the surface areas of objects such as engine cylinders, condensers, and pipes. Small areas are measured in square inches or square feet, and larger areas are measured in square yards, square rods, acres, and square miles.

Areas of Squares

22. Instead of finding the area of a square surface by counting the number of small squares contained by it, it is possible to find this area more easily simply by multiplying the length of the square by its width. Thus, referring to Fig. 7*(b),* there are 12 rows of little squares and 12 squares in each row, or 12 × 12 = 144 sq in. This is the result we would obtain by counting. When you use this method of multiplying the length by the width, the length and the width must be in the same units. In Fig. 7*(b)* both the length and the width are in inches, so the area is in square inches. In Fig. 7*(a)* the length and the width are given in feet, so the area is 1 ft × 1 ft = 1 sq ft.

Areas of Rectangles

23. Sometimes the area to be measured is a rectangle. This is a figure similar to the one shown in Fig. 8. It differs from a square because not all of its sides are of equal length. Only the sides that are opposite each other are equal. The area of a rectangle is found in the same way as the area of a square is found. That is, you multiply the length by the width. Again, both length and width must be expressed in the same units!

Thus, if a rectangle is 6 in. long and 4 in. wide, its area is 6 × 4 = 24 sq in.

This can be proved, as shown in Fig. 8(*b*), by dividing each side into inches, drawing lines across the figure from the points of division, and counting the number of squares thus formed. Each of these squares is 1 in. on each side, or 1 sq in. in area, and there are 24 squares in all. Hence the area of the rectangle is 24 sq in.

EXAMPLE 1. How many square inches are there in a rectangle 28 in. long and 13 in. wide?

SOLUTION. Multiply the length by the width; thus the area is $28 \times 13 = 364$ sq in. Ans.

EXAMPLE 2. A sheet of copper is 4 ft long and 3 in. wide. What is the area of its surface?

SOLUTION. Both dimensions must be expressed in the same units. When we reduce 4 ft to inches, the result is $12 \times 4 = 48$ in. The area, then, is $48 \times 3 = 144$ sq in. Ans.

When we reduce 3 in. to ft, the result is $3 \times \frac{1}{12} = \frac{1}{4}$ ft. Then, the area is $4 \times \frac{1}{4} = 1$ sq ft. Ans.

Note that the two answers, 144 sq in. and 1 sq ft, are equal.

EXAMPLE 3. The base of an office cabinet is $6\frac{1}{4}$ ft long and $2\frac{1}{2}$ ft wide. What area of floor space does it cover? Express the answer to the nearest tenth of a square foot.

SOLUTION. The area is the product of the two dimensions:

$$6\tfrac{1}{4} \times 2\tfrac{1}{2} = 6.25 \times 2.5 = 15.625, \text{ or } 15.6 \text{ sq ft} \quad \text{Ans.}$$

Check Your Learning

28. Flat surfaces are called __________ surfaces.

(plane)

29. In calculating the area of a surface, the dimensions must be expressed in the same __________.

(units)

30. A square has four equal sides and four right __________.

(angles)

31. The unit of area is a __________.

(square)

32. One square foot contains ________ sq in.

(144)

33. If the side of a square is 3 in., the area is ________ sq in.

(9)

34. The area of a rectangle is found by multiplying the __________ by the __________.

(length) (width)

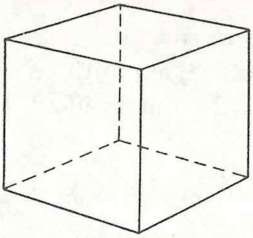


FIG. 9. CUBE

Practice Problems

1. How many square inches are there in the top of a desk 3 ft long and 27 in. wide?
2. A janitor wishes to wax the floor of a hall 30 yd long and 60 in. wide. How many square feet of floor space must he cover?
3. There are 12 panes of glass in a window. Each pane is 10″ by 14″. How many square inches of glass are there in each window?
4. A man wishes to paint both sides of a tight board fence which is 20 ft long and $6\frac{1}{2}$ ft high. How many square yards of surface will he paint?
5. A shopkeeper has four shelves along one side of his store. Each shelf is 20 ft long and 15 in. wide. How many square feet of shelf space does he have?
6. What is the area of a square plot of ground which is 90 ft on a side?
7. Which has the larger area, a square that is 10 ft on a side or a rectangle that is 20 ft long and 5 ft wide?

Meaning of Volume

24. *Volume* means the amount of space occupied by an object. For example, a block of wood like the one shown in Fig. 5 takes up a certain amount of space. That amount of space is the volume of the block. Likewise, the amount of space occupied by a certain quantity of water or other liquid is the volume of that liquid. And the amount of space occupied by a certain quantity of air or other gas is the volume of that gas.

The term "volume" is also used to indicate the amount of space inside a container. For example, the volume of a bottle is the amount of space in the bottle. The volume of the bottle is the same as the volume of liquid that the bottle will hold. The volume of a container is often called its *capacity* or its *cubic contents.*

Cubic Measure

25. A *solid* is a figure that has length, width, and thickness. The boundaries of a solid are called *surfaces*. A solid bounded by six squares, as shown in Fig. 9, is called a *cube*. If the solid is bounded by six rectangular surfaces, it is a *rectangular solid*. The space included between the bounding surfaces of a solid is called the cubic contents, capacity, or volume of the solid.

The unit of volume is a cube the edges of which are of a length equal to the corresponding unit of length. Thus, 1 cubic inch is the volume of a cube that measures 1 inch on each edge. Also, 1 cubic foot is the volume of a cube that

TABLE 4

UNITS OF CUBIC MEASURE

1728 cubic inches	= 1 cubic foot
27 cubic feet	= 1 cubic yard
128 cubic feet	= 1 cord
$16\frac{1}{2}$ to 25 cubic feet	= 1 perch

measures 1 foot on each edge, and so on. The abbreviation for cubic is cu; thus 1 cu in. is 1 cubic inch, 1 cu ft is 1 cubic foot, and so on. Some of the units of cubic measure are given in Table 4.

How Units of Volume Are Related

26. In Art. 18 you learned how to determine the number of square inches that are in a square foot. There you learned that, since $12^2 = 144$, there are 144 sq in. in 1 sq ft. It would be natural for you to suppose that the same reasoning would apply to cubic measures. In other words, you would think that, since $12^3 = 1728$, there are 1728 cu in. in a cubic foot. A glance at Table 4 of units of cubic measure will show you that your reasoning is correct. By using the same method, you can see that there are 27 cu ft in 1 cu yd because $3^3 = 27$. Thus we *cube* the number of smaller linear units that are in the larger linear unit to find the relation between the cubic measures. From this you should see why the third power of a number is called its cube.

Rule for Calculating Volume

27. The size of a solid is indicated by giving the dimensions of the solid connected by the word "by." For example, we would refer to a rectangular solid 3 ft wide, 2 ft deep, and 6 ft long as being 3 ft wide by 2 ft deep by 6 ft long.

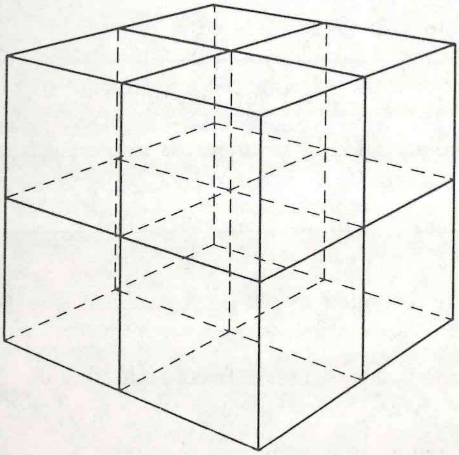


FIG. 10.

Frequently, this expression is shortened by using the symbol × for the word "by" and omitting the words "wide," "deep," and "long." Thus, we have 3 ft × 2 ft × 6 ft, where × is read "by."

The volume of any rectangular solid is the product of its three dimensions. For example, a box with inside dimensions 2 ft × 4 ft × 8 ft contains 2 × 4 × 8 = 64 cu ft. An iron bar 10 ft (or 10 × 12 in.) long, and with end surfaces that are each 1 in. square contains (10 × 12) × 1 × 1 = 120 cu in.

A cube 2 ft on each side is a 2-ft cube, and it contains 2 × 2 × 2 = 8 cu ft, as shown in Fig. 10. Be sure to note that there is a difference between the terms "cubic feet" and "feet cube" just as there is a difference between "square feet" and "feet square." In other words a 2-foot cube is not the same as 2 cubic feet.

To find the volume of a cube or a rectangular solid, multiply together the length, width, and depth, all expressed in the same units. If the dimensions are stated in inches, the volume will be expressed in cubic inches; if in feet, the volume will be given in cubic feet. If the dimensions are given in both feet and inches, they must be converted to the same units, either feet or inches, before any calculations are made.

EXAMPLE 1. What is the volume of a bin 14 ft long, 8 ft wide, and 6 ft high?

SOLUTION. Volume = length × width × depth

$$= 14 \times 8 \times 6 = 672 \text{ cu ft} \quad \text{Ans.}$$

EXAMPLE 2. How much metal is there in a block 4′6″ long, 8″ wide, and 2″ thick?

SOLUTION. *a.* Using dimensions expressed in feet,

$$\text{Volume} = 4\tfrac{1}{2} \times \tfrac{2}{3} \times \tfrac{1}{6} = \tfrac{9}{2} \times \tfrac{2}{3} \times \tfrac{1}{6} = \tfrac{1}{2}, \text{ or } 0.5 \text{ cu ft.} \quad \text{Ans.}$$

SOLUTION. *b.* Using dimensions expressed in inches,

$$\text{Volume} = 54 \times 8 \times 2 = 864 \text{ cu in.} \quad \text{Ans.}$$

Note the ½ cu ft is equal to 1728 ÷ 2 = 864 cu in.

Check Your Learning

35. The amount of space occupied by an object is its ____________.
(volume)

36. A figure that has length, width, and height is called a ____________.
(solid)

37. A rectangular solid is bounded by six ________________ surfaces.
(rectangular)

38. The volume of a cube that measures 1 ft on each edge is ____________.
(1 cu ft)

39. There are ________ cu in. in 1 cu ft.
(1728)

TABLE 5

UNITS OF LIQUID MEASURE

1 quart = 2 pints or 1 pint = $\frac{1}{2}$ quart
1 gallon = 4 quarts or 1 quart = $\frac{1}{4}$ gallon

1 U. S. gallon = 231 cubic inches
1 cubic foot = 7.481 U. S. gallons
1 imperial gallon = 277.42 cubic inches
1 liter = 1000 cubic centimeters
1 liter = 1000 milliliters

40. There are ________ cu ft in 1 cu yd.

(27)

Practice Problems

1. What is the volume of a cardboard carton 18″ long, 8″ wide, and 15″ high?
2. How much metal is there in five bars each of which is 2′3″ long, 6″ wide, and 3″ thick?
3. How many cubic yards of dirt could be removed by excavating a hole 50′ by 30′ by 10′?
4. What is the volume of a cube which is 3.5′ on an edge?

Units of Liquid Measure

28. There are special units for measuring the volume of liquids. Examples of liquids are water, milk, and oil. In the English system the names of the units commonly used for measuring liquids are the pint, the quart, and the gallon. The relations between these units are given in Table 5.

The gill (pronounced "jill") is used for measuring small amounts; the barrel is used for measuring large amounts. There are 4 gills in 1 pint, and 1 barrel contains $31\frac{1}{2}$ gallons. The hogshead, which is equal to 2 barrels, is rarely used nowadays.

The term "barrel" is sometimes used to refer to a container of a certain shape, even though its volume is not $31\frac{1}{2}$ gallons. That is why it is not always safe to assume that anything called a barrel holds $31\frac{1}{2}$ gallons. For example, the barrels used for crude oil and petroleum products hold 42 gallons each.

The abbreviations for the common units of liquid measure are pt for pint or pints, qt for quart or quarts, gal for gallon or gallons, and bbl for barrel or barrels.

The gallon used in the United States is not the same as the gallon used in Canada and Great Britain. The United States gallon has a volume of 231 cu in., and so there are 7.481 United States gallons in a cubic foot. The gallon used in Canada and Great Britain, called the *imperial gallon,* has a volume of 277.42 cu in. It is obvious that the imperial gallon is larger than the United States gallon.

TABLE 6

ENGLISH UNITS OF DRY MEASURE

1 peck = 8 quarts or 1 quart = $\frac{1}{8}$ peck
1 bushel = 4 pecks or 1 peck = $\frac{1}{4}$ bushel

1 U. S. bushel = 2150.42 cubic inches
1 imperial bushel = 2219.36 cubic inches

Since 277.42 ÷ 231 is about equal to 1.2, the imperial gallon is approximately 1.2 United States gallons. When we use the gallon without any modifying words, you are to assume that we mean the United States gallon.

In the metric system the basic unit for measuring liquids is the *liter*. Another smaller unit is the *milliliter*. There are 1000 milliliters in a liter. A liter has a volume of 1000 cu cm. These relations are listed in Table 5. The abbreviation l is used for liter or liters, and ml stands for milliliter or milliliters.

Units of Dry Measure

29. Certain things, such as fruits, vegetables, and grains, are measured in units of dry measure. In the English system these units are different from those used for liquid measure. In the metric system, however, the liter is used for both liquid and dry measure. The names of the common English units are the quart, the peck, and the bushel. There are 8 quarts in 1 peck, and there are 4 pecks in 1 bushel. The United States bushel contains 2150.42 cu in., and the imperial bushel contains 2219.36 cu in.

The relations between the English units of dry measure are summarized in Table 6. The abbreviations are qt for quart or quarts, pk for peck or pecks, and bu for bushel or bushels.

The dry quart is larger than the liquid quart. Since these two units are used for measuring different kinds of things, you should have no trouble knowing whether a liquid quart or a dry quart is meant when the term "quart" is used.

Other Units of Volume: Board Measure

30. A special kind of cubic measure is known as *board measure*. In measuring sawed lumber the unit of measure is the *board foot*. A board foot is equal to the volume of a board 1 ft square and 1 in. thick. A board foot, therefore, is equal to one-twelfth of a cubic foot. When the number of board feet in boards less than 1 in. thick is computed, the thickness is considered to be 1 inch. The abbreviation for board feet is fbm, which stands for "feet board measure." "Board feet" and "feet board measure" mean exactly the same thing, but the abbreviation used is always fbm.

For measuring wood the *cord* is sometimes used. A cord of wood is the amount of wood in a pile 8 ft long, 4 ft wide, and 4 ft high. Such a pile contains 128 cu ft. Hence a cord equals 128 cu ft.

To find the number of board feet in a piece of lumber, multiply together the length, in feet, the width, in inches, and the thickness, in inches, and divide the product by 12. Don't forget to consider thicknesses less than 1 inch as 1 inch.

EXAMPLE 1. How many board feet are there in a board 14 ft long, 8 in. wide, and 1 in. thick?

SOLUTION. The number of board feet is equal to

$$\frac{14 \times 8 \times 1}{12} = 9\tfrac{1}{3} \text{ fbm} \quad \text{Ans.}$$

EXAMPLE 2. How many board feet, to the nearest tenth, are there in a plank 10 ft 6 in. long, 15 in. wide, and 3 in. thick?

SOLUTION. Length = 10 ft 6 in. = $10\frac{1}{2}$ ft = 10.5 ft. The number of board feet is

$$\frac{10.5 \times 15 \times 3}{12} = 39.375, \text{ or } 39.4 \text{ fbm} \quad \text{Ans.}$$

EXAMPLE 3. Find the number of board feet (to the nearest tenth) in 20 pieces of siding $\frac{1}{2}$ in. thick, $5\frac{1}{2}$ in. wide, and 10 ft 9 in. long.

SOLUTION. Since the thickness is less than 1 in., the boards are considered to be 1 in. thick when finding the amount of material in them. Since 9 in. equals $\frac{3}{4}$ ft, the length = 10.75 ft, and the width = $5\frac{1}{2}$ in. = 5.5 in. The number of board feet is

$$\frac{20 \times 10.75 \times 5.5 \times 1}{12} = \frac{1182.5}{12} = 98.5 \text{ fbm} \quad \text{Ans.}$$

EXAMPLE 4. How many board feet of 2-in. planking are needed for a barn floor 18 ft × 25 ft?

SOLUTION. The planks are considered as 25 ft long. We first reduce the width of the floor to inches and consider the whole floor as one plank; the width is 18 × 12 = 216 in. Then, the number of board feet required is

$$\frac{25 \times 216 \times 2}{12} = 900 \text{ fbm} \quad \text{Ans.}$$

EXAMPLE 5. A board is $9\frac{1}{2}$ ft × 4 in. × $1\frac{1}{4}$ in. How many board feet does it contain?

SOLUTION. Following the same method as before, the result is

$$\frac{9\frac{1}{2} \times 4 \times 1\frac{1}{4}}{12} = 3.96 \text{ fbm} \quad \text{Ans.}$$

Check Your Learning

41. The pint, the quart, and the gallon are common units of measure for __________ .
(liquids)

42. The imperial gallon is (larger, smaller) than the United States gallon.
(larger)

43. The __________ is the basic unit of liquid and dry measure in the metric system.

(liter)

44. The quart, peck, and bushel are common units in English __________ measure.

(dry)

45. The unit of measure for sawed lumber is ________ ________ .

(board foot)

46. The abbreviation for board feet is _______ .

(fbm)

47. Four quarts is equal to one __________ .

(gallon)

48. There are _______ qt in a peck.

(8)

49. There are 1000 cu cm or 1000 ml in a __________ .

(liter)

50. A cord equals _______ cu ft.

(128)

Practice Problems

1. Change 20 gal to quarts.
2. How many pint bottles of milk can be filled from 4 gal of milk?
3. How many pecks of potatoes can a grocer make from 10 bushels of potatoes?
4. Change 3456 cu cm to liters.
5. How many pecks are there in 856 qt?
6. How many board feet are there in a board 10 ft 6 in. long, 9 in. wide, and $\frac{3}{4}$ in. thick?
7. Find the number of board feet, to the nearest tenth, in 12 pieces of siding $\frac{1}{2}$ in. thick, $4\frac{1}{2}$ in. wide, and 9 ft 6 in. long.

Measurement of Weight

What Weight Is

31. Every object is pulled toward the surface of the earth by the force of gravity. This pull is known as the *weight* of the object. The weight of an object depends on its volume and on the material of which it is composed. An object that has a great weight for a small volume is heavy; one that has a small weight for the same volume is light. For example, iron and concrete are considered to be heavy, and wood is considered to be light.

Avoirdupois Weight

32. In the English system the weights of most objects are measured in *avoirdupois* (pronounced "aver-do-poise") *units.* The most common unit is the *pound,* abbreviated lb. Large weights are measured in *tons,* and small weights are given in *ounces,* abbreviated oz. The ton generally used contains 2000 lb. Another kind of ton, still used for a few things such as railroad rails, contains 2240 lb. To distinguish between these two kinds of tons, the ton of 2000 lb is called the *short ton* and the ton of 2240 lb is called the *long ton.*

Another unit of avoirdupois weight you should know about is the *grain.* A pound contains 7000 grains. Since there are 16 oz in a pound, there are $7000 \div 16 = 437\frac{1}{2}$ grains in 1 oz. The unit called the *hundredweight* is sometimes used for steel bars and steel shapes for structural work. As the name implies, a hundredweight is 100 lb. The abbreviation for hundredweight is cwt.

The relations between the units of avoirdupois weight are shown in Table 7.

Other English Weights

33. For jewels and for precious metals, such as gold, silver, and platinum, troy weight is used. The names of the commonly used units in troy weight are the *grain,* the *pennyweight,* the *ounce,* and the *pound.* The troy grain is the same as the avoirdupois grain, and the troy ounce contains 480 grains. But since the troy pound contains 12 oz, it has only 5760 grains, whereas the avoirdupois pound has 7000 grains in its 16 oz. The troy weight units and equivalents are listed in Table 8.

In making up prescriptions, pharmacists use apothecaries' weight, Table 9. The common units are the *grain,* the *dram,* the *ounce,* and the *pound.* The apothe-

TABLE 7

AVOIRDUPOIS WEIGHT

1 pound	=	16 ounces
1 pound	=	7000 grains
1 short ton	=	2000 pounds
1 long ton	=	2240 pounds
1 hundredweight	=	100 pounds

TABLE 8

Troy Weight

1 pound = 12 ounces = 5760 grains
1 ounce = 20 pennyweight
1 pennyweight = 24 grains

caries' grain is the same as the avoirdupois grain and the troy grain. There are 60 grains in a dram; 8 drams in an ounce; and 12 oz in a pound. The abbreviation dr stands for dram or drams. Note the similarity between the troy pound and the apothecaries' pound. They are the same!

Metric Units of Weight

34. In the metric system the units of weight in common use are the *gram,* the *kilogram,* and the *milligram.* The gram contains 1000 milligrams, and the kilogram contains 1000 grams. An avoirdupois pound contains 453.6 grams, and 2.2046 avoirdupois pounds make a kilogram. Another unit that is used occasionally is the *metric ton,* which contains 1000 kilograms. The abbreviation g stands for gram or grams; mg stands for milligram or milligrams; and kg stands for kilogram or kilograms.

Gross Weight and Net Weight

35. The *gross weight* of an article means the weight of the article itself plus the weight of the package, crate, or vessel in which it is contained. The *net weight* means the weight of the article alone. For example, the gross weight of a reel of wire is the weight of the wire and reel together, and the net weight of the wire is the gross weight minus the weight of the reel.

Example 1. If a short ton of coal costs $16.00, how much does the coal cost per pound?

Solution. According to Table 7 there are 2000 lb in 1 ton. Since 1600 is the price of 2000 lb, it follows that the price of 1 lb is $\frac{1}{2000}$ of 1600 or $1600 \div 2000 = 0.8$, or $\frac{8}{10}$¢. If the price is to be expressed in dollars, the operation will be $16 \div 2000 = 0.008$, or $0.008. Ans.

Example 2. If a bag of sugar weighs 800 oz, what is its weight in pounds?

TABLE 9

Apothecaries' Weight

1 pound = 12 ounces = 5760 grains

1 ounce = 8 drams
1 dram = 60 grains

SOLUTION. According to Table 7, 1 lb = 16 oz. Therefore, dividing 800 by 16 will give the equivalent number of pounds. Thus,

$$800 \div 16 = 50 \text{ lb} \quad \text{Ans.}$$

EXAMPLE 3. How many grains are there in 1 lb of gold?

SOLUTION. According to Table 8, 1 lb troy = 12 oz, and 1 oz is equal to 20 pennyweight of 24 grains each. Hence, 1 lb is equal to

$$12 \times 20 \times 24 = 5760 \text{ grains} \quad \text{Ans.}$$

Check Your Learning

51. There are ________ oz in an avoirdupois pound.
(16)

52. There are 2000 lb in a ____________
(short ton or ton)

53. 1000 g make one ____________
(kilogram or kg)

54. 1000 mg make one ____________
(gram or g)

55. There are ________ oz in a troy pound.
(12)

56. A weight of 100 lb is called a ________________
(hundredweight)

57. The weight of an article plus the package or vessel in which it is contained is called the (gross, net) weight.
(gross)

58. The weight of the article alone is the (gross, net) weight.
(net)

Practice Problems

1. Change 35 avoirdupois pounds to ounces.
2. Change 38,964 lb to hundredweight.
3. An object weighs 5642 g. State the weight in kilograms.
4. How many grams are there in $2\frac{1}{4}$ lb?
5. A druggist prepares a capsule containing 5 grains. How many capsules could he fill from a dram?
6. A can of tomatoes contains 1 lb 2 oz of tomatoes. The can itself weighs 2 oz. Twenty-four of these cans are packed in a cardboard carton weighing $1\frac{1}{4}$ lb. Find the gross weight of the filled carton and the net weight of tomatoes.

Measurement of Time

Days and Years

36. The earth rotates on its axis. Also, the earth revolves around the sun. The amount of time required for the earth to make one turn on its axis is a *day,* and the amount of time required for the earth to make one journey around the sun is a *year.* Ten years is called a *decade,* and 100 years is called a *century.*

The *calendar year* contains exactly 365 days, but actually there are nearly 365¼ days in a *solar year.* To compensate for the difference between a solar year and a calendar year, the fractional parts of several years (most often 4 years) are added periodically to one year, which is then known as a *leap year.* A leap year contains 366 days; the extra day is added to February.

If the number expressing a year is divisible by 4, that year is a leap year, provided it is not also divisible by 100. A year that is divisible by 100 is a leap year only if the number expressing the year is also divisible by 400. Thus 1952, 1956, 1960, and 1964 were leap years, because each of those numbers is divisible by 4 and is not divisible by 100. The year 1900 was not a leap year, because 1900 is not divisible by 400 even though it is divisible by 100. The year 2000 will be a leap year, because 2000 is divisible by 400. Try these divisions for yourself.

Months and Weeks

37. In each year there are twelve *months.* April, June, September, and November each have 30 days. January, March, May, July, August, October, and December each have 31 days. February has 28 days in an ordinary year, but it has 29 days in a leap year.

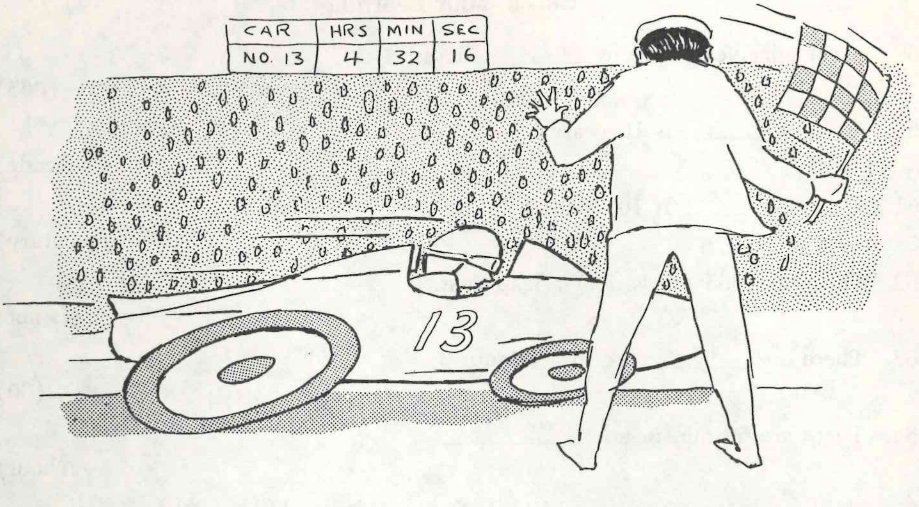
Thirty days hath September,
April, June, and November,
All the rest have thirty-one
Save February, which alone
Has twenty-eight and one day more
We add to it, one year in four.

A particular day is identified by the name of the month, the number of the day in the month, and the number of the year. An example is December 6, 1965, or 6 December 1965. When the number of the day is given by its name instead of by a digit or digits, the ordinal number is used. Thus, you should say or write "December sixth" or "the sixth of December."

Each group of seven days is a *week.* In a year consisting of 365 days, there are 52 weeks and one extra day.

Hours, Minutes, and Seconds

38. There are 24 *hours* in a day. An hour contains 60 *minutes,* and there are 60 *seconds* in a minute. "Minute" and "second" are also the names of units used to measure angles, but the units of time and the units of angle are, of course,



different kinds of units. When you use the terms "minutes" and "seconds" to give information to another person, be certain that he will know definitely whether you are talking about time or an angle. The abbreviation for hour or hours is hr; the abbreviation for minute or minutes is min; and the abbreviation for second or seconds is sec.

Time of Day

39. The 24 hours in a day are numbered from 1 to 12 in two separate sets. An ordinary watch or clock shows hours from 1 to 12. The time at the middle of the day is called 12 o'clock noon, and the time at the end of the day is called 12 o'clock midnight. To distinguish between the two sets of numbers of hours, the period before noon is known as A.M. time, and the period after noon is known as P.M. time. These abbreviations are also printed as a.m. and p.m.

To say what time it is, give the number of hours followed by the number of minutes past that hour. For example, at 25 minutes after 8 o'clock, the time is eight twenty-five. This is written as 8:25. Note the colon (:) used to separate the hour from the minutes. If necessary, A.M. or P.M. is written after the time, as in 1:45 P.M. At 12 o'clock noon the time is 12:00 A.M., and at 12 o'clock midnight it is 12:00 P.M.

The armed forces use another method of indicating the time of day. In this method the hours in a day are numbered continuously beyond 12. If the number of hours is less than 10, a cipher is written in front of the number. The hours of A.M. time are numbered up to 11, and the hours of P.M. time are numbered from 12 on. No colon is inserted between the number of hours and the number of minutes. For example, at 20 minutes after 8 o'clock in the morning, the time is written 0820 and is called "oh eight twenty." At 45 minutes after 2 o'clock in the afternoon, the time is written 1445 and is called "fourteen forty-five."

Check Your Learning

59. A calendar year contains ________ days.

(365)

60. A ____________ is 10 years.

(decade)

61. A ____________ is 100 years.

(century)

62. The year 1965 (is, is not) a leap year.

(is not)

63. There are ________ sec in a minute.

(60)

64. There are 60 min in an ____________.

(hour)

65. Twelve minutes after nine o'clock in the evening would be written ____________.

(9:12 P.M.)

66. Of the 12 months there are ________ months that have 30 days each.

(4)

Practice Problems

1. How many decades are there in a century?
2. Give the numbers of the leap years between 1972 and 2010.
3. How many days are there in 13 weeks?
4. Find the total number of days in June, July, and August.
5. My daughter was in the hospital 252 days. How many weeks was that?
6. How would nine minutes after one o'clock in the afternoon be written?
7. Write 23 minutes after 3 o'clock in the morning.
8. How many days are there in 72 hours?
9. In the armed forces what time is 1540?

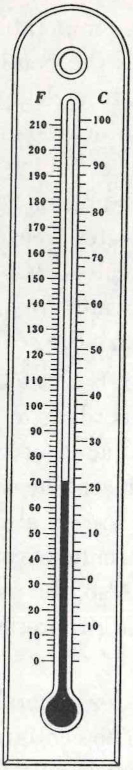


FIG. 11. THERMOMETER

Measurement of Temperature

Thermometer

40. The hotness or coldness of an object is indicated by its temperature. The temperature of a hot object is higher than the temperature of a cold object. However, the temperature of an object is not a measure of the amount of heat contained in the object.

Thermometers are the instruments used for measuring temperatures. A simple type of thermometer is shown in Fig. 11. It consists of a glass tube that is sealed airtight at the upper end and has, at its lower end, a bulb filled with mercury or alcohol. This tube is supported on a suitable frame, which also carries a graduated scale for the measurement of temperatures. As the temperature of the glass tube becomes higher, the liquid in the tube expands and its top surface rises. As the temperature becomes lower, the surface of the liquid drops. In this manner the position of the liquid surface on the scale shows the temperature of the glass tube and therefore the temperature of the surroundings.

Thermometer Scales

41. Two systems for measuring temperature are in common use. One is the Fahrenheit system, and the other is the centigrade system. In both systems the unit of measure is a *degree*. However, a degree in one system does *not* represent the same change in temperature as a degree in the other system represents. In the Fahrenheit system the temperature at which water freezes is called 32 degrees and the temperature at which water boils is called 212 degrees. In the centigrade system the temperature at which water freezes is called 0 degrees and the temperature at which water boils is called 100 degrees. These two temperatures, namely, the temperature at which water freezes and the temperature at which it boils, are the two points used to mark the scales on the thermometer.

On the thermometer in Fig. 11 two scales are shown. The one at the left, with F at the top, is graduated according to the Fahrenheit system. The one at the right, with C at the top, is graduated according to the centigrade system. On each scale the small divisions represent degrees and the graduation marks at 0 and at each multiple of 10 degrees are numbered. Note that the graduation mark representing 32 degrees on the Fahrenheit scale is opposite the mark representing 0 degrees on the centigrade scale. Also, the mark representing 212 degrees on the Fahrenheit scale is opposite the mark representing 100 degrees on the centigrade scale.

The Fahrenheit system is used for general purposes in the United States and other English-speaking countries. The centigrade system is used in scientific work in the United States and other English-speaking countries and for all purposes in many countries.

Degrees of Temperature

42. In the Fahrenheit system the number of degrees between the temperature at which water boils and the temperature at which water freezes is the difference between 212 and 32. This difference is 180. So a degree in the Fahrenheit system, or a *degree Fahrenheit,* represents the change in temperature obtained by determining the total change from 32 degrees to 212 degrees and imagining that it consists of 180 equal parts. Degrees of the same size are used to measure temperatures above 212 degrees and below 32 degrees.

In the centigrade system the number of degrees between the temperature at which water boils and the temperature at which water freezes is the difference between 100 and 0. This difference is 100. So a degree in the centigrade system, or a *degree centigrade,* represents the change in temperature obtained by determining the total change from 0 degrees to 100 degrees and imagining that it consists of 100 equal parts. Degrees of the same size used to measure temperatures above 100 degrees and below 0 degrees.

As you probably know, in many parts of the world the temperature of the air falls below 0 degrees on the Fahrenheit scale, and in many more places it falls below 0 degrees on the centigrade scale. Temperatures below 0 degrees (Fahrenheit or centigrade) can be produced in several commercial processes. Temperatures above 212 degrees on the Fahrenheit scale and above 100 degrees on the centigrade scale are produced in ovens and furnaces. A temperature below 0 degrees is often indicated by the term "minus" or the symbol −. Thus a temperature of 20 degrees below zero is written minus 20 degrees or −20 degrees.

The symbol ° or the abbreviation deg may be used instead of the word "degree" or "degrees" after a number representing a temperature. Also, to show whether the temperature is Fahrenheit or centigrade, the abbreviation F or C is generally used. For example, a temperature of 20 degrees Fahrenheit may be written 20 deg F or 20°F. However, when the abbreviation F or C is used, the symbol ° or the abbreviation deg is usually omitted. Thus 20 F means 20 degrees Fahrenheit and 10 C means 10 degrees centigrade.

As explained in Art. 10, the degree is also the name of a unit for measuring angles, and the symbol ° is also used to indicate degrees of angle. However, the degree of temperature and the degree of angle are not related in any way. So when the word "degree" or the symbol ° is used, you should have no trouble knowing whether the unit of temperature or the unit of angle is meant.

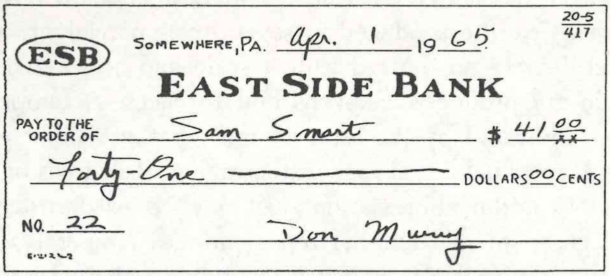
Check Your Learning

67. A thermometer is used to measure ________.
(temperature)

68. In the Fahrenheit system, water freezes at ________ and boils at ________.
(32°) (212°)

69. In the centigrade system, water freezes at ________ and boils at ________.
(0°) (100°)

70. A temperature written as −20° means 20° below ________.
(0°)



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Forty-One DOLLARS 00 CENTS

NO. 22

Don Murry

Other Units of Measure

Money

43. In both the United States and Canada, the base of the money, or monetary, system is the *dollar*. Also, in both countries a dollar is worth 100 *cents*, or a cent is $\frac{1}{100}$ dollar. The symbol $ in front of a number stands for dollars. Thus, $20 is read twenty dollars. The symbol ¢ after a number stands for cents. Thus, 25¢ is read twenty-five cents.

An amount of money less than a dollar may be written either as a decimal of a dollar or as a number of cents. For example, 22 cents may be indicated either as $0.22 or as 22¢. When an amount of money is expressed in dollars and cents, it is usually written as a mixed decimal, with the number of cents shown as a decimal part of a dollar. Thus five dollars eighteen cents is usually written $5.18, and eleven dollars two cents is usually written $11.02. The digit in the second decimal place is always written, even if it is a cipher. For example, five dollars fifty cents, or five dollars and a half, is written $5.50. It is never written $5.5.

Sometimes it is necessary to express an amount of money in dollars to more than two decimal places. For example, the average cost of one of a large number of similar objects may be given as $1.2463. This amount would usually be read one point two four six three dollars.

The unit called the *mill* is sometimes used in monetary expressions. The mill is equal to a thousandth of a dollar. Thus $0.025 is read twenty-five mills, instead of two point five cents or two and one-half cents. Mills are commonly used in stating tax rates.

Miscellaneous Units

44. In counting sheets of paper, the terms *quire* and *ream* are used. A quire contains 24 sheets, and a ream consists of 20 quires. The abbreviation qr stands for quire or quires, and rm stands for ream or reams.

Many articles are sold by the *dozen*. There are 12 things in a dozen. Thus a dozen eggs consists of 12 eggs. Also, 12 dozen makes a *gross*. The abbreviation doz stands for dozen, and gr stands for gross.

Other terms that are sometimes used to refer to quantities are the *pair* and

"Four score and seven years-"

the *score*. A pair consists of two things, usually similar things. For instance, one speaks of a pair of shoes or a pair of gloves. The abbreviation pr stands for pair. A score contains 20 things. A common use is in the expression "a score of years," which means 20 years. Thus when Lincoln said, "Four score and seven years," he meant 87 years.

Check Your Learning

71. The base of our monetary system is the ____________.

(dollar)

72. One cent is ________ dollar.

$(\frac{1}{100})$

73. There are 24 sheets of paper in a ____________.

(quire)

74. There are ________ eggs in a dozen.

(12)

75. A ____________ means 20 things.

(score)

Practice Problems

1. Write each of the following amounts of money by using the symbol $ and figures:
   a) Seven dollars twenty-five cents
   b) Ten dollars ten cents
   c) Fifty-four dollars thirteen cents
   d) Two dollars six cents
2. Write each of the following amounts of money in words:
   a) $1.11 *b*) $6.40 *c*) $25.75 *d*) $0.05
3. A teacher wishes to give each student in his class three sheets of paper for use in the final examination. If there are 42 people in the class, how many quires of paper must he have?
4. Pam Conway wishes to make articles for a bake sale. She will use 29 eggs. How many boxes containing a dozen eggs each must she buy at the grocery store?
5. The school tax in a certain district is 47 mills. Express this in decimal parts of a dollar.
6. James Calpin gave his age as three score years and ten. How old is he?

Conversion of Units

Types of Problems in Conversion

45. In practical work you often have a quantity expressed in one kind of unit that you want to express in another kind of unit. For example, you have a distance in feet and you want to express it in inches or in meters. Or you may be given a volume in cubic feet which you are required to express in cubic yards or in gallons.

In another type of problem a quantity expressed in two or more kinds of unit must be expressed in units of one kind. For example, you may know a weight in pounds and ounces and have to express it in pounds alone or in ounces alone. Or you may know time in hours, minutes, and seconds and have to express it in hours alone, minutes alone, or, for that matter, seconds alone.

A third type of problem is the reverse of the type just described. In this type a quantity expressed as a number of units of one kind has to be expressed in units of more than one kind. For instance, an angle in degrees and a decimal part of a degree has to be expressed in degrees and minutes or in degrees, minutes, and seconds. Or, perhaps, a distance in inches is to be expressed in feet and inches.

Remember that in any problem such as the problems outlined in the preceding paragraphs the same quantity is expressed in two different ways. Therefore, the unit or units given in the result must measure the same kind of quantity as the given unit or units. Thus, you can change, or convert, feet to yards because both the foot and the yard are units of length. Also, you can convert cubic feet to gallons because both the cubic foot and the gallon are units of volume. Remember, however, that the gallon is used only for measuring liquids, but the cubic foot may be used for measuring solids and gases as well as liquids.

Changing denominate numbers from one kind to another is called *converting* or *reducing* them. Thus you can say that you have converted 3 ft to 1 yd or that you have reduced 3 ft to 1 yd. Don't forget that the reduction of denominate numbers is the process of changing their denomination *without changing their value.* Denominate numbers can be changed in two ways. In one way the change is to a lower unit, as from feet to inches. In the other way, the change is to a higher unit, as from inches to feet. The first example is called *reduction descending.* The second example is *reduction ascending.* Thus when you change 2 miles to 10,560 ft, you apply the reduction descending process. And when you change 243,080 lb to 121.54 tons, you apply the reduction ascending process. In each case, however, we say that the numbers have been *reduced.*

Changing to Units of Smaller Size

46. When you change a denominate number to a smaller unit, you will get more of the smaller units. Therefore, multiplication is the key to reducing units of a higher denomination to units of a lower denomination. As we said before, this reduction is known as reduction descending. To reduce descending, multiply

the given number by the number of units of the lower denomination required to make one unit of the higher denomination. For example, to change 3 ft to inches, multiply the number of given units, in this case 3, by 12 because 12 of the smaller unit, the inch, make one of the larger unit, the foot. The answer, of course, is 36 in. As another example, to reduce 2 hr to minutes, multiply 2 by 60, because 1 hr contains 60 min. Thus, 2 hr = 120 min. Or to reduce $4\frac{1}{2}$ yd to feet, multiply $4\frac{1}{2}$ by 3 because there are 3 ft in a yard. This gives $4\frac{1}{2} \times 3 = 13\frac{1}{2}$. Thus, $13\frac{1}{2}$ ft equals $4\frac{1}{2}$ yd.

EXAMPLE 1. Express 8.62 ft in inches. Round off your answer to one decimal place.

SOLUTION. Since 1 ft contains 12 in., find the number of inches in 8.62 ft by multiplying 8.62 by 12. Thus $8.62 \times 12 = 103.44 = 103.4$ and the required distance is then 103.4 in. Ans.

EXAMPLE 2. How many minutes are there in an angle of 4.7°?

SOLUTION. A degree of angle contains 60 min, so find the number of minutes in the given angle by multiplying 4.7 by 60. The product is 282.0. Hence $4.7^\circ = 282.0'$. Ans.

EXAMPLE 3. The weight of a certain object is given as 24.9 kg. What is its weight in pounds expressed to the nearest hundredth?

SOLUTION. The number of pounds in 1 kg is 2.2046. Therefore, the weight of the object in pounds is 24.9×2.2046. This product is 54.89454, or 54.89 lb. Ans.

EXAMPLE 4. Express 0.641 ft in inches and a fraction of an inch. Round off the result to the nearest $\frac{1}{16}$ in.

SOLUTION. When the given decimal of a foot is multiplied by 12, the product is 7.692, so 0.641 ft equals 7.692 in. You must now change 0.692 to a fraction having 16 as a denominator. To find the numerator, multiply 0.692 by 16 and round off the product to the nearest whole number. Since $0.692 \times 16 = 11.072$, the required fraction is $\frac{11}{16}$. Thus 0.641 ft equals $7\frac{11}{16}$ in. Ans.

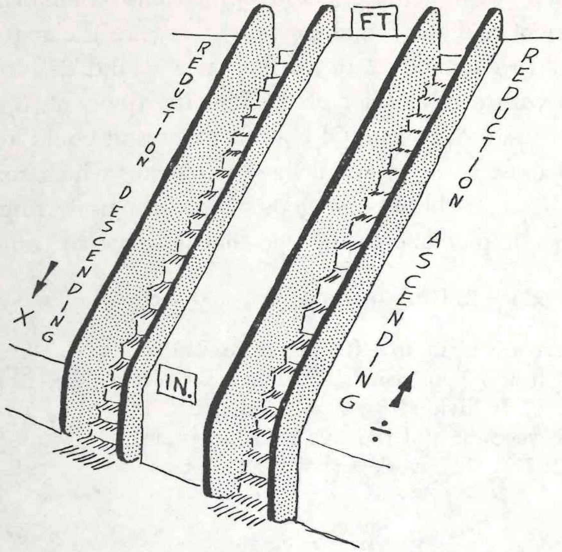
EXAMPLE 5. How many seconds are there in a day?

SOLUTION. Since a day contains 24 hr and an hour contains 60 min, the number of minutes in a day is 24×60. Also, since a minute contains 60 sec, the number of seconds in a day is $24 \times 60 \times 60$. This product is 86,400. So there are 86,400 sec in a day. Ans.

Check Your Learning

76. Changing quarts to gallons is an example of reduction (ascending, descending). (ascending)

77. Changing gallons to quarts is an example of reduction (ascending, descending). (descending)



78. To change 3 hr to minutes, ____________ 3 by 60.

(multiply)

79. To change 10 gal to quarts, ____________ 10 by 4.

(multiply)

Practice Problems

1. The area of a certain portion of the surface of a pavement is 32.14 sq yd. What is the area of that portion in square feet to the nearest tenth?
2. The volume of a certain container is given as 0.375 cu ft. What is the volume in cubic inches?
3. Change 0.817 ft to inches and a fraction of an inch; take the result to the nearest $\frac{1}{8}$ in.
4. How many centimeters are there in 3.16 ft? HINT. First convert the given number of feet to inches.

Changing to Units of Larger Size

47. If reduction to a lower denomination is performed by multiplying, it is reasonable to think that reduction to a higher denomination is done by dividing. Actually, dividing *is* the key to changing units of a lower denomination to units of a higher denomination. This change, or reduction, is known as reduction ascending. To reduce ascending, divide the given number by the number of units of the lower denomination unit required to make one unit of the higher denomination. For example, to change 32 oz (avoirdupois) to pounds, divide the number of units, 32, by 16 because there are 16 oz in 1 lb. Thus 32 ÷ 16 = 2 lb.

Quite often when a denominate number is reduced to a higher denomination, the division will not be exact. When this happens only the quotient will be of the higher denomination; *the remainder will be of the same denomination as the given number.* The quotient and the remainder, together, are the required result. For example, if you had to reduce 42 in. to feet, you would divide 42 by 12. The result would be 3 with a remainder of 6. Here the quotient, 3, is feet and the remainder, 6, is inches, or 3 ft 6 in. Of course, this result could also be expressed as $3\frac{1}{2}$ or 3.5 ft. In most cases you will have to decide which form is better for the particular problem. If there is no good reason for preferring one form over another, choose the simpler one or the one that is easier to compute.

EXAMPLE 1. Reduce 211 in. to yards, feet, and inches.

SOLUTION. There are 12 in. in 1 ft. Therefore, 211 in. divided by 12 = 17 ft and 7 in. remainder. There are 3 ft in 1 yd. Therefore, 17 ft divided by 3 = 5 yd and 2 ft remainder. The last quotient and the two remainders are the answer, 5 yd 2 ft 7 in. The work is shown at the right.

17 ft
12)211
12
91
84
7 in. remainder

3)17
5 yd 2 ft remainder

EXAMPLE 2. Reduce 3533 pt to gallons and quarts.

SOLUTION. There are 2 pt in 1 qt. Therefore, 3533 pt = 1766 qt and 1 pt remainder. There are 4 qt in 1 gal. Therefore, 1766 qt = 441 gal and 2 qt remainder.

2)3533
4)1766 qt 1 pt remainder
441 gal 2 qt remainder

Hence, 3533 pt equals 441 gal 2 qt 1 pt. Ans.

EXAMPLE 3. Reduce 18,000 sq in. to square feet.

SOLUTION. Divide 18,000 by the number of square inches in 1 sq ft.

125 sq ft Ans.
144)18000
144
360
288
720
720

EXAMPLE 4. How many weeks and days are there in 18 days?

SOLUTION. It takes 7 days to make 1 week; that is, it takes 7 units of the lower denomination to make 1 unit of the higher denomination. Therefore, 18 has to be divided by 7 to get the result. Thus,

7)18
2 weeks and 4 days remainder

The result is therefore 2 weeks 4 days. Ans.

EXAMPLE 5. Reduce 14,728 lb to tons.

SOLUTION. One ton is equal to 2000 lb; thus, 14,728 is to be divided by 2000:

$$\begin{array}{r l} & \underline{7} \text{ tons} \\ 2000\overline{)14728} & \\ \underline{14000} & \\ 728 & \text{lb remainder} \end{array}$$

Therefore, 14,728 lb = 7 tons 728 lb. Ans.

EXAMPLE 6. Reduce 985 pt (dry measure) to higher denominations.

SOLUTION. The solution is found in the same manner as in Example 2. The final quotient and the various remainders are 15 bu 1 pk 4 qt 1 pt. Hence the final answer is 15 bu 1 pk 4 qt 1 pt. Ans.

Further along in this text you'll find some practice problems. Be sure that you solve each one of them. They will give you the practice you need in applying the preceding methods. Use them to test how well you have mastered the principles we have just explained.

Check Your Learning

80. To change 12 qt to gallons, ____________ 12 by 4.
(divide)

81. To change 32 qt to pecks, ____________ 32 by 8.
(divide)

82. Reduce 576 sq in. to sq ft. Answer is ________ sq ft.
(4)

Practice Problems

1. Reduce the following to units of higher denominations. If fractions appear in the results in any but the lowest denomination, reduce them to the lowest denomination.
   a) 7460 sq in. *b*) 2019 pt (liquid)
2. How many tons and pounds are there in 459,875 lb?
3. If 36 cu ft of coal weighs 1 ton, how much space, to the nearest cubic foot, is filled by 7 tons 1590 lb?
4. How many rails, each 30 ft long, will be required to lay a railroad track 26 miles long?
5. Reduce:
   a) 4 rods 2 yd 2 ft to feet *b*) 4 bu 3 pk 2 qt to quarts
6. A student purchased 8 lb 5 oz (avoirdupois) of chemicals for experimental work. What was the total cost at 8¢ an ounce?
7. The bearings of a machine need 3 pt of oil each day. How many quarts do they need in a 5-day week?
8. The pressure of the atmosphere at sea level is, at a given time, 14.7 pounds per square inch. To the nearest pound, what is the pressure in pounds per square foot?
9. Express the answer to Problem 8 in tons. Round off your answer to two decimal places.

Changing Compound Numbers to Smaller Units of One Kind

48. Sometimes when a quantity is expressed in two kinds of units, you may want to express it as a number of units of the smaller kind. To do this, you pro-

ceed as follows: First, change the given number of units of the larger size to a number of units of the desired smaller size. Then add to this result the given number of units of the smaller size.

In studying the following solutions, you should note carefully what is done when it is necessary to multiply and add in the same problem.

EXAMPLE 1. A train required 4 hr 18 min to travel from one city to another. How many minutes did it require for the trip?

SOLUTION. First change 4 hr to minutes. Do this by multiplying 4 by 60, because there are 60 min in 1 hr. The product is 240, so the number of minutes in 4 hr is 240. To find the number of minutes in 4 hr 18 min, add the 18 to the 240 just found. The sum of 240 and 18 is 258; therefore, the time is 258 min. Ans.

The work may be indicated as follows:

$$4 \times 60 + 18 = 240 + 18 = 258 \text{ min}$$

Note that you must multiply the 4 by the 60 before you add the 18. Another way to arrange the work is shown at the right.

$$\begin{array}{r} 4 \times 60 = 240 \\ 18 \\ \hline 258 \end{array}$$

EXAMPLE 2. A distance is given as 6 ft $3\frac{1}{4}$ in. What is the distance in inches?

SOLUTION. First change 6 ft to inches by multiplying 6 by 12. The product is 72, so the number of inches in 6 ft is 72. To find the number of inches in the given distance, add the $3\frac{1}{4}$ to the 72 just found. The result is $75\frac{1}{4}$. Thus 6 ft $3\frac{1}{4}$ in. is equal to $75\frac{1}{4}$ in. Ans.

The work may be indicated in the following manner:

$$6 \times 12 + 3\tfrac{1}{4} = 72 + 3\tfrac{1}{4} = 75\tfrac{1}{4} \text{ in.}$$

Changing Compound Numbers to Larger Units of One Kind

49. Sometimes a quantity may be expressed in two kinds of unit and you may want to express it as a number of units of the larger given kind. In a problem of this type the given number of units of the smaller size is usually less than the number of such units in a unit of the larger size. First, change the given number of units of the smaller size to a fraction or a decimal of a unit of the larger size. Then add this fraction or decimal to the given number of units of the larger size.

EXAMPLE 1. Change 4 hr 18 min to hours.

Solution. First change 18 min to a part of an hour by dividing 18 by 60. You can indicate the division by writing the fraction $\frac{18}{60}$. When you reduce this fraction to lowest terms by dividing both the numerator and the denominator by 6, you get $\frac{3}{10}$. However, you would usually find it better to express 18 min as a decimal of an hour. The quotient obtained by dividing 18 by 60 is 0.3, so the given period of time is equal to either $4\frac{3}{10}$ or 4.3 hr. Ans.

You should note that the given number of hours is not changed in this example.

EXAMPLE 2. Change 6 ft $3\frac{1}{4}$ in. to feet. Round off the result to the nearest hundredth of a foot.

SOLUTION. First change $3\frac{1}{4}$ in. to a decimal part of a foot by dividing $3\frac{1}{4}$ by 12. To carry out the division, you should change $\frac{1}{4}$ to a decimal. Thus $3\frac{1}{4} = 3.25$. When 3.25 is divided by 12 and the quotient is rounded off to two decimal places, the result is 0.27. So $3\frac{1}{4}$ in. is equal to 0.27 ft. When you add this decimal of a foot to 6 ft, you get 6.27 ft. Ans.

You should note that, again, the given number of feet is not changed. You should also note that $3\frac{1}{4}$ in. is equal to 3.25 in. and is not equal to 0.325 ft. Do not make the mistake of calling the result 6.325 ft. Be sure to change 3.25 in. to 0.27 ft. Then add this decimal to 6 ft.

Practice Problems

1. Change 6 min 37 sec to seconds.
2. Change 3 lb $5\frac{1}{2}$ oz avoirdupois to ounces.
3. An angle is given as $7°14'$. What is the angle in minutes?
4. Change 15 ft 9 in. to feet.
5. Change 8 ft $2\frac{7}{8}$ in. to feet. Express the result to the nearest hundredth of a foot.
6. The standard distance for the marathon race in the Olympic games is 26 miles 385 yd. What is this distance in miles?

More Changes in Denominate Numbers

50. It may happen that a quantity is expressed in three different kinds of unit and you want to express it as a number of units of one of the three kinds. The solution of a problem of this type is just an application of the principles of the preceding articles, as shown in the following examples.

EXAMPLE 1. An angle is given as $2°28'37''$. Express the angle in seconds.

SOLUTION. A good way to solve this example is as follows: First change $2°28'$ to minutes. Thus the number of minutes in $2°28'$ is

$$2 \times 60 + 28 = 120 + 28 = 148$$

Then change $148'37''$ to seconds. The number of seconds in $148'37''$ is

$$148 \times 60 + 37 = 8880 + 37 = 8917 \text{ sec} \quad \text{Ans.}$$

Another way to solve this example is to change 2° to seconds, then to change $28'$ to seconds, and finally to find the sum of these two numbers of seconds and $37''$. The work may be indicated as follows:

$$\begin{aligned} 2 \times 60 \times 60 &= 7200'' \\ 28 \times 60 &= 1680'' \\ &\underline{37''} \\ \text{Sum} &= 8917'' \quad \text{Ans.} \end{aligned}$$

EXAMPLE 2. Express the angle in Example 1 in degrees.

SOLUTION. One way to solve this example is to change $28'37''$ to minutes, then to change this number of minutes to a decimal of a degree, and, finally, to add this decimal to 2°.

To change $37''$ to a decimal of a minute, divide 37 by 60. The quotient, rounded off to two figures (because 37 has two figures), is 0.62. So $28'37'' = 28.62'$.

To change 28.62′ to a decimal of a degree, divide by 60. The quotient, rounded off to four figures, is 0.4770. Hence, 2°28′37″ = 2.4770° Ans.

EXAMPLE 3. Express the angle in Example 1 in minutes.

SOLUTION. To solve this example, you must change 2° to minutes and change 37″ to a decimal of a minute. You can then add these numbers of minutes to 28′. The work may be arranged as follows:

$$\begin{array}{rr} 2 \times 60 = & 120' \\ & 28' \\ 37 \div 60 = & \underline{0.62'} \\ \text{Sum} = & 148.62' \quad \text{Ans.} \end{array}$$

Changing to Units of More than One Kind

51. We shall now show you how to solve problems that are the reverse of the type described in the preceding article; that is, the quantity is expressed in one kind of unit and you are to express it in two or more kinds of unit. You will be able to understand the general instructions for solving such problems better if you first see how some examples are worked.

EXAMPLE 1. Express 17.864° in degrees and minutes.

SOLUTION. The desired number of degrees is the integral part of the given number, or 17. Find the desired number of minutes by changing the decimal part of a degree to minutes. Since 1° contains 60′, multiply 0.864 by 60. The product, rounded off to three figures, is 51.8. So 17.864° is equal to 17°51.8′ Ans.

EXAMPLE 2. Change 4.692 ft to feet and inches. Round off the result to the nearest $\frac{1}{16}$ in.

SOLUTION. The desired number of feet is 4, which is the integral part of the given distance. To change the decimal part of a foot to inches, multiply 0.692 by 12. The product, rounded off to three figures, is 8.30. Now you must change the decimal of an inch to a fraction having 16 as the denominator. Find the numerator by multiplying 0.30 by 16 and rounding off the product to the nearest whole number. Since 0.30 × 16 = 4.8 and the nearest whole number is 5, the fraction is $\frac{5}{16}$. So 0.692 ft equals $8\frac{5}{16}$ in., and 4.692 ft equals 4 ft $8\frac{5}{16}$ in. Ans.

Summary of Conversions

52. The work of the preceding article can be summarized as follows:

When a quantity is expressed in units of one kind and you want to express it in units of that kind and also units of a smaller size, take the following steps:

Step 1. Use the integral part of the given number as the number of units of the given kind in the desired result.

Step 2. Change the decimal part of the given number to units of the desired smaller size. Express a part of the smaller unit either as a decimal or as a fraction with a suitable denominator.

EXAMPLE. Change 8.156 miles to miles and feet.

SOLUTION. The integral number of miles in the distance is 8. Since 1 mile contains 5280 ft, you can find the number of feet by multiplying 0.156 by 5280. When a decimal part of a unit is changed to a number of units of smaller size, it is usually necessary to decide how to round off the number of smaller units.

In this example the given number is carried to thousandths of a mile. Since one thousandth of a mile is about 5 ft, the number of feet should be rounded off to the nearest 5. The product of 0.156 and 5280 is 823.68. So the distance should be taken as 8 miles 825 ft. Ans.

If the given distance were 8.056 miles, the number of feet corresponding to 0.056 mile also would be rounded off to the nearest 5 ft. Since $0.056 \times 5280 = 295.68$, the distance would be taken as 8 miles 295 ft.

If the given distance were 8.15 miles, the result would be rounded off differently. One hundredth of a mile is about 53 ft, so the number of feet would be rounded off to the nearest 50. Since $0.15 \times 5280 = 792$, the distance would be taken as 8 miles 800 ft.

53. If a quantity is expressed in one kind of unit, you may want to express it in units of both a larger size and the given kind. For example, you may want to reduce 37 ft to yards and feet. In a reduction such as this, the number of the larger units will be a whole number. That is, in this example when you reduce 37 ft to yards and feet, the number of yards will be 12, a whole number. For this type of problem you can use the following method.

Step 1. Divide the given number of units by the number of units of this kind in one unit of the larger size, just as if you were going to change to units of the larger size alone. See Art. 47.

Step 2. Stop the division when the decimal point in the quotient is reached. In other words, find only the whole number in the quotient.

Step 3. Take the whole number in the quotient as the desired number of units of the larger size, and take the last difference as the desired number of units of the given kind left over.

If the given number contains a decimal or a fraction, you should ignore the decimal or the fraction when carrying out the division, and you should include it in the last difference in step 3 of the method.

EXAMPLE 1. Express an angle of 87.2′ in degrees and minutes.

SOLUTION. First ignore the decimal and find the number of degrees by dividing 87 by 60. Since the whole number in the quotient is 1, there is 1°. Then the number of minutes left over is the difference between 87.2 and 60, or 27.2. So $87.2' = 1°27.2'$. Ans.

EXAMPLE 2. Change $202\frac{1}{4}$ in. to feet and inches.

SOLUTION. Ignore the fraction in the given number and divide 202 by 12 until the decimal point is reached. The work is shown at the right. The desired number of feet is 16. Since the last difference is 10 and the fraction of an inch in the given distance is $\frac{1}{4}$, the desired number of inches is $10\frac{1}{4}$. So $202\frac{1}{4}$ in. equals 16 ft $10\frac{1}{4}$ in. Ans.

$$\begin{array}{r l} & \underline{16}\text{ ft} \\ 12\big) & 202 \\ & \underline{12} \\ & 82 \\ & \underline{72} \\ & 10\text{ in. remainder} \end{array}$$

54. Occasionally, a quantity will be expressed in units of one kind and you will want to express it in units of two larger sizes and units of the given kind. You can first apply the method explained in Art. 53 to change the given number of units to units of a second size and units of the given kind. Then you can apply the same method again to change the computed number of units of this second size to units of the largest size and units of the second size. You should not forget to include in the final result the number of units of the given kind left over from the first division.

EXAMPLE. An angle is given as 7413″. What is the angle in degrees, minutes, and seconds?

SOLUTION. First change 7413″ to minutes and seconds by dividing by 60. The division is shown at the right. You get 123′33″.

Now change 123′ to degrees and minutes by dividing 123 by 60. The whole number in the quotient is 2; the product of 2 and 60 is 120; and the difference obtained by subtracting 120 from 123 is 3. So, 123′ = 2°3′. The required angle is 2°3′33″. Ans.

Note that the 33″ from the first division must be added to the 2°3′ obtained by the second division.

$$\begin{array}{r l} & \underline{123}\text{ min} \\ 60\big) & 7413 \\ & \underline{60} \\ & 141 \\ & \underline{120} \\ & 213 \\ & \underline{180} \\ & 33\text{ sec remainder} \end{array}$$

Practice Problems

1. Change 3 hr 26 min 27 sec to minutes.
2. Express the amount of time in Problem 1 in hours.
3. The length of a steel bar is given as $40\frac{5}{8}$ in. What is the length of the bar in feet and inches?
4. An angle is given as 97.3°. What is the angle in degrees and minutes?
5. Express an angle of 11,755″ in degrees, minutes, and seconds.

Conversion of Temperatures

55. A special explanation is needed to change a Fahrenheit temperature to a centigrade temperature or to change a centigrade temperature to a Fahrenheit temperature. From the information in Arts. 41 and 42, you should see that the following two statements are true:

1. A change in temperature of 100 degrees centigrade corresponds to a change of 180 degrees Fahrenheit.

2. A temperature of 0 C is the same as a temperature of 32 F.

Since 180 divisions on the Fahrenheit scale are equivalent to 100 divisions on the centigrade scale, it follows that 1 degree centigrade is equal to $\frac{9}{5}$ degrees Fahrenheit. (Perhaps you better refer back to Fig. 11.) From this relation and the fact that 0 F is 32 Fahrenheit degrees below the mark for 0 C, the following rules are derived:

RULE I. To find the Fahrenheit temperature, multiply the centigrade temperature by $\frac{9}{5}$ and add 32° to the product.

RULE II. To find the centigrade temperature, subtract 32° from the Fahrenheit temperature and multiply the remainder by $\frac{5}{9}$.

Example 1. What is the Fahrenheit equivalent of 85 C?

SOLUTION. By Rule I, the temperature is

$$\left(85 \times \tfrac{9}{5}\right) + 32 = 185\text{ F} \quad \text{Ans.}$$

EXAMPLE 2. If a temperature is 68 F, what is it in degrees centigrade?

SOLUTION. According to Rule II, 32 is first subtracted from 68, and the remainder is then multiplied by $\frac{5}{9}$:

$$68 - 32 = 36 \qquad 36 \times \tfrac{5}{9} = 20\text{ C} \quad \text{Ans.}$$

Converting Variations of Temperature

56. You must make a clear distinction between temperature and a change of temperature when you convert from one scale to the other. When a variation or a change in temperature is converted from one scale to the other, 32° is *not* considered. For example, suppose that the temperature of an electric generator rises 40 degrees centigrade and the change in degrees Fahrenheit is required. Since 1 degree centigrade = $\frac{9}{5}$ degrees Fahrenheit, the change in degrees Fahrenheit is $\frac{9}{5} \times 40 = 72°$. Again, suppose that the temperature decreases from 180 F to 99 F, a change of 81 degrees Fahrenheit, and it is desired to express this change or variation in degrees centigrade. Since 1 degree Fahrenheit = $\frac{5}{9}$ degree centigrade, the variation in degrees centigrade is $\frac{5}{9} \times 81 = 45°$.

EXAMPLE 1. The temperature of the air in a room increases from 43 F to 88 F. What is the change expressed in degrees centigrade?

SOLUTION. First, find the change in temperature in degrees Fahrenheit by subtracting 43 from 88. The difference is 45 degrees. Since a change of 1 degree Fahrenheit is equal to a change of $\frac{5}{9}$ degree centigrade, a change of 45 degrees Fahrenheit is equal to $45 \times \frac{5}{9}$, or 25, degrees centigrade. Ans.

EXAMPLE 2. During a certain process, the temperature of the gas in a container is reduced from 90 C to 15 C. Find the change in degrees Fahrenheit.

SOLUTION. The drop in temperature is equal to 90 − 15, or 75, degrees centigrade. Since a change of 1 degree centigrade is equal to a change of $\frac{9}{5}$ degrees Fahrenheit, the change of 75 degrees centigrade is equal to $75 \times \frac{9}{5} = 135$ degrees Fahrenheit. Ans.

Be sure to note that you do *not* consider the number 32 when converting a *change* in temperature from one scale to another.

Practice Problems

1. What is the equivalent of 76 C on the Fahrenheit scale?
2. If a temperature is 80 F, what is the equivalent on the centigrade scale?
3. The temperature of outdoor air dropped from 57 F to 32 F. What was the change, expressed in degrees centigrade?
4. The temperature in a cooker increased from 112 F to 250 F. What was the change expressed in degrees centigrade?
5. During a certain process, the temperature of the gas in a container is reduced from 98 C to 10 C. Find the change in degrees Fahrenheit.

Calculations with Measured Quantities

Types of Problems to Be Considered

57. At this point you should know how to change any quantity expressed in certain units to an equal quantity expressed in any other kind of unit.

We shall now show you how to solve other problems:

1. Problems in which you have to find the sum of two or more quantities expressed in units of more than one kind.
2. Problems in which you have to find the difference between two quantities expressed in units of more than one kind.
3. Problems in which you have to find the product of a quantity expressed in units and another such quantity or a number without units.
4. Problems in which you have to find the quotient obtained by dividing a quantity expressed in units by another such quantity or by a number without units.

Addition of Quantities

58. In solving a problem of the first type, you may want to find the sum of two or more quantities each of which is expressed in terms of more than one kind of unit. For instance, you may want to add 3 ft $6\frac{1}{2}$ in. to 4 ft $2\frac{1}{4}$ in. or you may want to find the sum of 20°15′ and 32°48′. One method of solving the first example is to change each distance to inches and then find the sum of the two distances in inches. Thus 3 ft $6\frac{1}{2}$ in. equals $42\frac{1}{2}$ in.; 4 ft $2\frac{1}{4}$ in. equals $50\frac{1}{4}$ in.; and the sum of $42\frac{1}{2}$ in. and $50\frac{1}{4}$ in. is $92\frac{3}{4}$ in. If you wish to express the sum in feet and inches, you may write it as 7 ft $8\frac{3}{4}$ in.

You can solve the second example quite easily by changing each angle to degrees and a decimal of a degree and then adding the two numbers thus obtained. Since 20°15′ equals 20.25° and 32°48′ equals 32.80°, the desired result is the sum of 20.25° and 32.80°. This sum is 53.05°. If you wish to express the result in degrees and minutes, you will get 53°3′.

But there is an easier way to solve the examples of the preceding paragraphs. To solve the first example, you can simply add the given numbers of feet to get the number of feet in the sum, and you can add the given numbers of inches to get the number of inches in the sum. The sum of 3 and 4 is 7, and the sum of $6\frac{1}{2}$ and $2\frac{1}{4}$ is $8\frac{3}{4}$. Therefore, the sum of 3 ft $6\frac{1}{2}$ in. and 4 ft $2\frac{1}{4}$ in. is 7 ft $8\frac{3}{4}$ in. This result is the same as the one we obtained by the other method.

When you add the given numbers of degrees in the second example, or find the sum of 20° and 32°, you get 52°. And when you add the given numbers of minutes, or find the sum of 15′ and 48′, you get 63′. The sum of 20°15′ and 32°48′ found in this way is 52°63′. But 63′ is equivalent to 1°3′, so 52°63′ is the same as the sum of 52° and 1°3′. This sum is 53°3′, which was found previously.

To find the sum of two or more quantities each of which is expressed in terms of more than one kind of unit, proceed in this way:

Step 1. Arrange the given quantities so that the numbers of units with the same name are in columns and then find the sum of the numbers in each column.

Step 2. If the sum of the units of one kind is greater than the number of such units in one unit of the next larger size, express that sum in terms of the two kinds of units. Then carry the number of units of the larger size thus found to the column for such units, and retain the number of smaller units left over.

EXAMPLE 1. A workman spent 1 hr 40 min on one job, 2 hr 35 min on a second job, and 3 hr 10 min on a third job. How much time did he spend on the three jobs?

SOLUTION. Arrange the three given times as shown at the right, with the numbers of hours in one column and the numbers of minutes in another column. The sum of the numbers of hours is 6, and the sum of the numbers of minutes is 85. Since there are 60 min in 1 hour and 85 is greater than 60, change 85 min to 1 hr 25 min. Then add the 1 hr to the 6 hr to get 7 hr and take the 25 min left over as the number of minutes. The sum is 7 hr 25 min. Ans.

$$\begin{array}{rrl} & 1\text{ hr} & 40\text{ min} \\ & 2 & 35 \\ & 3 & 10 \\ \hline & 6\text{ hr} & 85\text{ min} \\ \text{or} & 7\text{ hr} & 25\text{ min} \quad \text{Ans.} \end{array}$$

EXAMPLE 2. You want to order one steel bar that can be cut into four parts having these lengths: 1 ft $3\frac{1}{2}$ in., 2 ft $7\frac{3}{4}$ in., 1 ft $11\frac{1}{8}$ in., and 3 ft $8\frac{3}{8}$ in. If the length lost at each cut is ignored, what is the required length of the bar?

SOLUTION. Place the numbers of feet and inches in columns as at the right. When the numbers in the columns are added, the sums are 7 ft and $30\frac{3}{4}$ in. However, $30\frac{3}{4}$ in. is equal to 2 ft $6\frac{3}{4}$ in. So add the 2 ft to the 7 ft to get 9 ft, and take the $6\frac{3}{4}$ in. left over as the number of inches. The required length of the bar is 9 ft $6\frac{3}{4}$ in. Ans.

$$\begin{array}{rrl} & 1\text{ ft} & 3\frac{1}{2}\text{ in.} \\ & 2 & 7\frac{3}{4} \\ & 1 & 11\frac{1}{8} \\ & 3 & 8\frac{3}{8} \\ \hline & 7 & 30\frac{3}{4}\text{ in.} \\ \text{or} & 9\text{ ft} & 6\frac{3}{4}\text{ in.} \quad \text{Ans.} \end{array}$$

Subtraction of Quantities

59. To show you how to find the difference between two quantities each of which is expressed in terms of more than one kind of unit, we shall describe the procedure for subtracting 1 ft $8\frac{3}{4}$ in. from 4 ft $6\frac{1}{2}$ in.

First, arrange the given quantities so that the numbers of units with the same name are in columns and so that the quantity with the larger number in the left-hand column is above the quantity with the smaller number in that column. Since the number of feet in 4 ft $6\frac{1}{2}$ in. is greater than the number of feet in 1 ft $8\frac{3}{4}$ in., write the first quantity above the second one, as shown in Fig. 12.

Now you are ready to subtract. Starting with the column at the right, try to subtract the lower number from the upper number. If the lower number is smaller than the upper number, find the difference and write it as the number of units of the proper name in the required result. If the lower number is greater than the upper number, as in this example, you must do this: Reduce the upper number in the next column to the left by 1, and increase the upper number in the right-

$$\begin{array}{ll} 4\text{ ft} & 6\tfrac{1}{2}\text{ in.} \\ 1 & 8\tfrac{3}{4} \\ \hline & \end{array} \qquad \begin{array}{ll} 3\text{ ft} & 18\tfrac{1}{2}\text{ in.} \\ 1 & 8\tfrac{3}{4} \\ \hline & \end{array} \qquad \begin{array}{ll} 3\text{ ft} & 17\tfrac{6}{4}\text{ in.} \\ 1 & 8\tfrac{3}{4} \\ \hline 2\text{ ft} & 9\tfrac{3}{4}\text{ in.} \end{array}$$

(a) (b) (c)

FIG. 12. SUBTRACTION

hand column by the number of units of the kind in the right-hand column contained in one unit of the kind in the column to the left. In this example subtract 1 from the number 4 in the left-hand column. Then, since 1 ft is equal to 12 in., add 12 to the upper number in the right-hand column. The quantity 4 ft 6½ in. thus becomes 3 ft 18½ in., as shown in Fig. 12*(b)*.

You can find the number in the right-hand column in the required difference by subtracting the lower number from the new upper number in the usual way. Replace the number 18½ by $17\frac{6}{4}$, as shown in Fig. 12*(c)*. The difference between $17\frac{6}{4}$ and 8¾ is 9¾. This is the number of inches in the required result. Write is below the subtraction line.

Find the number of feet in the required result by subtracting 1 ft from 3 ft. The complete quantity obtained by subtracting 1 ft 8¾ in. from 4 ft 6½ in. is 2 ft 9¾ in.

EXAMPLE 1. You have a piece of pipe 8 ft 6 in. long, and you plan to cut off a piece 2 ft 9¼ in. long. If no allowance is made for the length lost by the cut, what will be the length of the piece of pipe that is left?

$$\begin{array}{lll} 8\text{ ft }6\text{ in.} = 7\text{ ft} & 17\tfrac{4}{4}\text{ in.} & \\ \qquad\qquad\quad 2 & 9\tfrac{1}{4} & \\ \hline \qquad\qquad\quad 5\text{ ft} & 8\tfrac{3}{4}\text{ in.} & \text{Ans.} \end{array}$$

SOLUTION. Before you make the cut, you can compute the length that will be left by subtracting 2 ft 9¼ in. from 8 ft 6 in. You can find the difference in the manner indicated at the right. When you look at the given quantities, you see at once that you cannot start the subtraction until you make certain changes in the quantity 8 ft 6 in. When you make these changes, 8 ft 6 in. becomes 7 ft $17\frac{4}{4}$ in. Then subtract 9¼ from $17\frac{4}{4}$ to get 8¾ in., and subtract 2 from 7 to get 5 ft. The desired length is 5 ft 8¾ in. Ans.

EXAMPLE 2. You want to meet a train that is expected to arrive at the railroad station at 5:16 P.M. You estimate that you will need 1 hr 20 min to drive from your home to the station, and you want to allow about 15 min extra for unexpected delays and parking. At what time should you leave your home?

$$\begin{array}{ll} 5\text{ hr }16\text{ min} = 4\text{ hr} & 76\text{ min} \\ \qquad\qquad\qquad\quad 1 & 35 \\ \hline \qquad\qquad\qquad\quad 3\text{ hr} & 41\text{ min} \end{array}$$

SOLUTION. First you should find the sum of the time needed for traveling and the time allowed for delays and parking by adding the 15 min to the 1 hr 20 min. This sum is 1 hr 35 min. You can then determine when you should leave your home by subtracting 1 hr 35 min from the time at which the train is expected to arrive at the station. Since 5:16 P.M. is 5 hr 16 min after noon, you should subtract 1 hr 35 min from 5 hr 16 min. Since 16 is less than 35, you should change 5 hr 16 min to 4 hr 76 min before subtracting. As shown at the

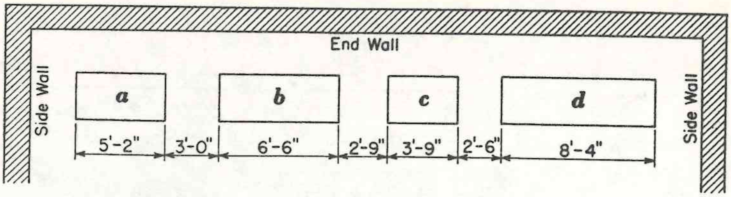


FIG. 13. DIMENSIONS IN PROBLEMS 1 AND 2

right, the difference between 4 hr 76 min and 1 hr 35 min is 3 hr 41 min. This means that you should leave home 3 hr 41 min after noon, or at 3:41 P.M. Ans.

Practice Problems

1. You are planning to place four machines in a row in a factory. The widths of the machines *a*, *b*, *c*, and *d*, and the required distances between them, are given by the dimensions indicated in Fig. 13. What is the distance, in feet, from the left-hand side of machine *a* to the right-hand side of machine *d*?

2. The distance between the side walls of the building in Fig. 13 is 38 ft 4 in. If you so locate machine *a* that the distance from its left-hand side to the wall at the left is 2 ft $8\frac{1}{4}$ in., what will be the distance from the right-hand side of machine *d* to the wall at the right?

3. You have a piece of lumber 10 ft $6\frac{3}{4}$ in. long and you want to saw off two pieces having lengths of 2 ft $4\frac{1}{2}$ in. and 3 ft $9\frac{3}{4}$ in. If no allowance is made for the length lost by the saw cuts, what is the length of the piece that would be left?

4. You plan to start on a trip in your car at 7:15 A.M. If you estimate that the actual driving time required for the trip will be 6 hr 40 min and that you will stop for a total of 45 min for lunch and rest, at what time should you plan to complete the trip?

Multiplying Denominate Numbers

60. Sometime you may be asked to find the product of two quantities each of which is expressed in feet and inches. For example, you may want to find the area of a square 3 ft 4 in. on a side. To do this, you must multiply the given distance (3′4″) by itself, so you must first express this distance in feet alone or in inches alone. Thus, one method of finding the square of 3 ft 4 in. is to take it as 40 in. and find the square of 40. Since $40 \times 40 = 1600$, the result is 1600 sq in. Another method is to take 3 ft 4 in. as $3\frac{1}{3}$ ft. To square $3\frac{1}{3}$, you must first reduce it to $\frac{10}{3}$ and then square $\frac{10}{3}$. The result is $\frac{100}{9}$ sq ft. You may, of course, express 3 ft 4 in. in this example in feet and a decimal of a foot and then square the mixed decimal. This method, however, requires more time and work than either of the other methods.

A similar problem will arise if you are asked to find the area of a rectangle. Assume, as an example, that the dimensions of the rectangle are 6 ft $1\frac{1}{2}$ in. and 2 ft 9 in. First you must change both given distances to inches alone or change both dimensions to feet alone. Do not change one distance to feet and the other to inches. Here is one way to find the product of the given distances: Change 6 ft $1\frac{1}{2}$ in. to $73\frac{1}{2}$ or 73.5 in. and change 2 ft 9 in. to 33 in. Then multiply 73.5 by 33 to get 2425.5, and call the result 2426 sq in. Another way to find the product is this: First change 6 ft $1\frac{1}{2}$ in. to 6.125 ft and change 2 ft 9 in. to 2.75 ft. Then multiply 6.125 by 2.75 to get 16.84375, and call the result 16.84 sq ft. As a check, compare the two results.

Never try to find the square of 3 ft 4 in. by squaring 3 ft to get 9 sq ft, squaring 4 in. to get 16 sq in., and then adding these squares to get 9 sq ft 16 sq in. This result is not correct. Nor should you try to find the product of 6 ft $1\frac{1}{2}$ in. and 2 ft 9 in. by multiplying 6 ft by 2 ft to get 12 sq ft, multiplying $1\frac{1}{2}$ in. by 9 in. to get 13.5 sq in., and adding these values to get 12 sq ft 13.5 sq in. This result, too, is incorrect.

Multiplying Denominate and Abstract Numbers

61. In another type of problem you will need to multiply a quantity expressed in more than one unit by an abstract number. One way to find the product is as follows: Express the quantity given in two or more units as a number in units of one kind, and then multiply this number by the abstract number. For example, to multiply 1 ft $3\frac{1}{8}$ in. by 16, change 1 ft $3\frac{1}{8}$ in. to $15\frac{1}{8}$ or $\frac{121}{8}$ in. and then multiply this fraction by 16 to get 242 in.

There is another method of finding the product of a quantity expressed in more than one kind of unit and a number without units. We shall show you the method by working some examples.

EXAMPLE 1. You need 16 steel bars each 1 ft $3\frac{1}{8}$ in. long. What is the total length of the bars?

SOLUTION. The total length is the product of 1 ft $3\frac{1}{8}$ in. and 16. First multiply 1 ft by 16 to get 16 ft and then multiply $3\frac{1}{8}$ in. by 16 to get 50 in. Next change 50 in. to 4 ft 2 in. and add this distance to the 16 ft. Since the sum of 16 ft and 4 ft 2 in. is 20 ft 2 in., the required length is 20 ft 2 in. Ans.

The calculations may be indicated as follows:

$$\begin{aligned} 1\text{ ft} \times 16 &= 16\text{ ft} \\ 3\tfrac{1}{8}\text{ in.} \times 16 &= 50\text{ in.} = 4\text{ ft } 2\text{ in.} \\ 16\text{ ft} + 4\text{ ft } 2\text{ in.} &= 20\text{ ft } 2\text{ in.} \end{aligned}$$

Note that the product of 1 ft $3\frac{1}{8}$ in., or $15\frac{1}{8}$ in., and 16 is 242 in.

EXAMPLE 2. The average time needed by a workman to make an article is 3 hr 25 min. How much time will he need to make six articles?

SOLUTION. You find the total time by multiplying 3 hr 25 min by 6. The calculations are as follows:

$$\begin{aligned} 3\text{ hr} \times 6 &= 18\text{ hr} \\ 25\text{ min} \times 6 &= 150\text{ min} = 2\text{ hr } 30\text{ min} \\ 18\text{ hr} + 2\text{ hr } 30\text{ min} &= 20\text{ hr } 30\text{ min} \quad \text{Ans.} \end{aligned}$$

Dividing Denominate Numbers

62. There will be times when you will have to solve a problem in division in which both the dividend and the divisor are denominate numbers. For example, you may be asked to divide 7 hr 30 min by 1 hr 25 min or to divide 16 ft 8 in. by 2 ft 2½ in. You can solve a problem of this type in this way: First express each of the given quantities in units of the same kind. Then divide the dividend by the divisor in the usual way. We shall illustrate the procedure by solving some examples.

EXAMPLE 1. Assume that you work 7 hr 30 min each day and that the time needed to make an article is 1 hr 25 min. How many articles can you make in a day, and how much extra time will you have in a day?

SOLUTION. You can find the number of complete articles by dividing 7 hr 30 min by 1 hr 25 min, using the integral part of the quotient. You can find the amount of extra time by considering the last difference obtained in the division after you get the integral part of the quotient.

In order to do the division, express 7 hr 30 min as 450 min and express 1 hr 25 min as 85 min. The division is shown at the right. Since the integral part of the quotient is 5, you can make five complete articles. Ans.

$$\begin{array}{r} 5 \\ 85\overline{)450} \\ \underline{425} \\ 25 \end{array}$$

In this example the dividend and the divisor are whole numbers. So the number of minutes not needed for making the five complete articles will be the difference 25. Thus the amount of extra time will be 25 min. Ans.

When the dividend or the divisor contains a decimal part, you must consider the position of the decimal point in the remainder, as shown in the next example.

EXAMPLE 2. You have a steel bar 16 ft 8 in. long, and you want to know how many pieces 2 ft 2½ in. long you can cut from it. You also want to know the length of the piece that will be left.

SOLUTION. To find the number of pieces 2 ft 2½ in. long, divide 16 ft 8 in. by 2 ft 2½ in. Use the integral part of the quotient. To find the length of the piece that will be left, consider the last difference obtained in the division.

Before trying to divide, express 16 ft 8 in. as 200 in. and express 2 ft 2½ in. as 26.5 in. Then perform the division as indicated at the right. Since there is one decimal place in the divisor, move the decimal point one place to the right in both the divisor and the dividend. The integral part of the quotient is 7. Thus you can get seven pieces 2 ft 2½ in. long from the given bar. Ans.

$$\begin{array}{r} 7 \\ 26.5\overline{)200.0} \\ \underline{1855} \\ 145 \end{array}$$

In this example we moved the decimal to perform the division. Thus 26.5 was changed to 265 and 200 was changed to 2000. The whole number 7 in the quotient was not affected by these changes, but the remainder was affected. The product of 26.5 and 7 is 185.5, and the difference obtained by subtracting 185.5 from 200 is 14.5. On the other hand, the product of 265 and 7 is 1855, and the difference obtained by subtracting 1855 from 2000 is 145. Obviously you can't omit the decimal point in the remainder if you want to know how many inches of a bar 200 in. long will be left after seven pieces each 26.5 in. long have been cut from it. Therefore, you must locate the decimal point in the last difference in line with the decimal point in the original dividend, 200. Place the decimal point in the last difference, or remainder, between the 4 and the 5. Thus, the length after the seven pieces have been cut off will be 14.5, or $14\frac{1}{2}$ in., or 1 ft $2\frac{1}{2}$ in. Ans.

Dividing a Denominate Number by an Abstract Number

63. Another type of problem in division involves dividing a quantity expressed in units of more than one kind by an abstract number. One method of doing this is to express the dividend as a number of units of one kind and then divide in the usual manner. For example, suppose you have a steel bar 28 ft 2 in. long and you want to cut it into eight equal parts. You can find the length of each part by dividing 28 ft 2 in. by 8. Express 28 ft 2 in. as 338 in. The quotient obtained by dividing 338 by 8 is 42.25; therefore, the length of each part should be 42.25 in. This result can be expressed as $42\frac{1}{4}$ in. or as 3 ft $6\frac{1}{4}$ in.

Another method of obtaining this answer is illustrated in the following example:

EXAMPLE 1. A steel bar 28 ft 2 in. long is to be cut into eight equal parts. What should be the length of each part?

SOLUTION. First divide the number of feet in the given length by 8, and use the whole number in the quotient as the number of feet in the desired length of each part. Thus, when 28 is divided by 8, the whole number in the quotient is 3; so the number of feet in the length of each part is 3.

To find the number of inches in the length of each part, first subtract 8 × 3 ft from 28 ft. Thus $28 - 24 = 4$ ft. Express this difference in inches; add to it the 2 in. in the length of the original bar; and divide this sum by 8. The difference 4 ft is equal to 48 in. Add 48 and 2 to get 50, and divide 50 by 8. The result is $6\frac{1}{4}$, so the length of each part is 3 ft $6\frac{1}{4}$ in. Ans.

$$\begin{array}{l} \quad\ \ 3\text{ ft} \\ 8\overline{)28} \\ \quad\ \underline{24} \\ \quad\ \ \ 4 \times 12 = 48 \\ 48 + 2 = 50\text{ in.} \\ 50 \div 8 = 6.25 = 6\tfrac{1}{4} \end{array}$$

The calculations are shown to the right of the explanation.

EXAMPLE 2. The total time required by a workman to make six articles was 11 hr 24 min. What was the time needed for making one article?

SOLUTION. You can find the time for making one article by dividing 11 hr 24 min by 6. The whole number in the quotient obtained by dividing 11 by 6 is 1, and 5 hr is left over. To find the number of minutes in the desired time, express 5 hr in minutes. Add the 24 min in the total time to that result and divide the sum by 6. Thus 5 hr equals 300 min; the sum of 300 and 24 is 324; and the quotient obtained by dividing 324 by 6 is 54. So the time to make one article is 1 hr 54 min. Ans.

The calculations may be indicated as follows:

$$\begin{array}{l} \quad\ \ 1\text{ hr} \\ 6\overline{)11} \\ \quad\ \ \underline{6} \\ \quad\ \ 5 \times 60 = 300 \end{array} \qquad\qquad \begin{array}{l} 300 + 24 = 324\text{ min} \\ 324 \div \ \ 6 = \ \ 54\text{ min} \end{array}$$

The same result could be obtained in this manner:

$$11 \text{ hr } 24 \text{ min} = 11.4 \text{ hr}$$
$$11.4 \div 6 = 1.9$$
$$1.9 \text{ hr} = 1 \text{ hr } 54 \text{ min} \quad \text{Ans.}$$

Practice Problems

1. Find the square of 6 ft 3 in. Express the result *a*) in square inches and *b*) in square feet to the nearest square foot.
2. What is the product of 7 ft $2\frac{1}{2}$ in. and 4 ft 7 in.? *a*) Express the result in square inches. Round off the result to the nearest whole number. *b*) Express the result in square feet. Round off the result to the nearest tenth.
3. You have 12 similar objects, each weighing 3 lb $4\frac{1}{2}$ oz. What is the total weight of the 12 objects, in pounds and ounces? Find the result first by changing 3 lb $4\frac{1}{2}$ oz to ounces and then without making this change.
4. You have a piece of lumber 15 ft 8 in. long, and you want to cut it into pieces each of which is 2 ft $4\frac{1}{4}$ in. long. *a*) How many pieces of this length can you get? *b*) What is the length of the piece that will be left? Ignore the part of the length lost in cutting.
5. Suppose that you want to cut the piece of lumber in Problem 4 into six parts of equal length. What should be the length of each part, in feet and inches? Round off the result to the nearest eighth of an inch.

Answers to Practice Problems

Article 11

1. 9°
2. 23°45′
3. 2°35′14″
4. 2700′
5. 36°30′
6. 16°22′5″
7. 2460″

Article 14

1. 153 ft
2. 120 in.
3. 100 yd
4. 52,800 ft
5. 600 cm
6. 7.62 cm
7. 6 miles
8. 264 ft long; 495 ft wide
9. 984.25 in.
10. 6.3 m

Article 23

1. 972 sq in.
2. 450 sq ft
3. 1680 sq in.
4. $28\frac{8}{9}$ sq yd
5. 100 sq ft
6. 8100 sq ft
7. They are equal.

Article 27

1. 2160 cu in.
2. 2430 cu in. or $1\frac{13}{32}$ cu ft
3. $555\frac{5}{9}$ cu yd
4. 42.875 cu ft

Article 30

1. 80 qt
2. 32 pt
3. 40 pecks
4. 3.456 liters
5. 107 pecks
6. 7.9 fbm
7. 42.8 fbm

Article 35

1. 560 oz
2. 389.64 cwt
3. 5.642 kg
4. 1020.6 grams
5. 12 capsules
6. 31 lb 4 oz gross; 27 lb net

Article 39

1. 10
2. 1976, 1980, 1984
   1988, 1992, 1996
   2000, 2004, 2008
3. 91 days
4. 92 days
5. 36 weeks
6. 1:09 P.M.
7. 3:23 A.M.
8. 3 days
9. 3:40 P.M.

Article 44

1. *a*) $7.25
   b) $10.10
   c) $54.13
   d) $2.06
2. *a*) One dollar eleven cents
   b) Six dollars forty cents
   c) Twenty-five dollars seventy-five cents
   d) Five cents
3. $5\frac{1}{4}$ quires
4. 3 boxes; Pam needs $2\frac{5}{12}$ doz
5. $0.047
6. 70 years

Article 46

1. 289.3 sq ft
2. 648 cu in.
3. $9\frac{6}{8}$ in.
4. 96.3 cm

Article 47

1. *a*) 5 sq yd 6 sq ft 116 sq in.
   b) 8 bbl 1 qt 1 pt
2. 229 tons 1875 lb
3. 281 cu ft
4. 9152 rails
5. *a*) 74 ft
   b) 154 qt
6. $10.64
7. 7.5 qt
8. 2117 lb per sq ft
9. 1.06 tons per sq ft

Article 49

1. 397 sec
2. $53\frac{1}{2}$ oz
3. 434′
4. 15.75 ft
5. 8.24 ft
6. 26.22 miles

Article 54

1. 206.45 min
2. 3.44 hr
3. 3 ft $4\frac{5}{8}$ in.
4. 97°18′
5. 3°15′55″

Article 56

1. **$168\frac{4}{5}$ F**
2. **$26\frac{2}{3}$ C**
3. $13\frac{8}{9}$ C
4. $76\frac{2}{3}$ C
5. **$158\frac{2}{5}$ F**

Article 59

1. 32 ft
2. 3 ft $7\frac{3}{4}$ in.
3. 4 ft $4\frac{1}{2}$ in.
4. 2:40 P.M.

Article 63

1. *a*) 5625 sq in.
   b) 39 sq ft
2. *a*) 4758 sq in.
   b) 33.0 sq ft
3. 39 lb 6 oz
4. *a*) 6 pieces
   b) 1 ft $6\frac{1}{2}$ in.
5. 2 ft $7\frac{3}{8}$ in.

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Practical Arithmetic

Serial 6750E
PART 5
Edition 1

Examination Questions

Notice to Students.—Study this instruction text thoroughly before you answer the following questions. Read each question carefully and be sure you understand it; then write the best answer you can. If the answer involves a mathematical solution, show enough of your work to indicate how you obtained your answer. We will not accept answers alone. When you complete your work, examine it closely, correct all the errors you can find, and see that every question is answered; then mail your work to us. DO NOT HOLD IT until another examination is ready.

The first 8 questions are worth 10 points each while Questions 9, 10, 11, and 12 are worth 5 points each.

Give answers only for the first eight questions.

1. Convert and express your answer to two decimal places.
   a) 5.65 gal = 22.60 qt
   b) 342 sq in. = 2.38 sq ft
   c) 384 cu ft = 14.22 cu yd
   d) 90 cm = 2.95 ft
   e) A change of 64 Fahrenheit degrees equals a change of 35.56 centigrade degrees.

2. *a)* 2 yd = 72 in.
   b) 40 articles = 3.33 doz
   c) 7600 lb = 3.8 short tons
   d) 3 days = 72 hr
   e) 1 bu 3 pk = 7 pk

3. *a)* 36 sq ft = 4 sq yd
   b) 3 gross = 3 articles
   c) ¾ sq mi = 480 acres
   d) 400 sq mm = 4 sq cm
   e) 3½ cords = 348 cu ft

4. *a)* 4 bbl = 126 gal
   b) 45 C = 113 F
   c) 5 sq yd = 45 sq ft
   d) 2 gal 1 qt = 17 pt
   e) 54 cu ft = 2 cu yd

5. *a)* 3 cu ft = 5184 cu in.
   b) 30 min = 1800 sec
   c) 86 F = 30 C
   d) 63.5 cm = 25 in.
   e) 5°6′3″ = 18363 sec

6. *a)* 320 in. = 8 yd 2 ft 8 in.
   b) 74 qt = 2 bu 1 pk 2 qt
   c) 0.675° = 40 min 30 sec
   d) 6240 lb = 3 short tons 240 lb
   e) 31500 sec = 52 hr 30 min

7. Perform the following operations:
   a) 20°30′23″ × 4 = 82° 1′ 32″
   b) 8 yd 2 ft $4\frac{1}{2}$ in. × 5 = 43 yd 2 ft $10\frac{1}{2}$ in.
   c) 359°10′40″ ÷ 7 = 51° 18′ 40″
   d) 10 yd 1 ft 8 in. ÷ 4 = 2 yd 1 ft 11 in.
   e) 3 pk 6 qt ÷ 2 = 1 pk 7 qt

8. Add:

(a)	(b)	(c)
2 yd 1 ft 6 in.	1 ft $8\frac{1}{2}$ in.	3° 4′ 8″
3 yd 2 ft 2 in.	6 ft $1\frac{3}{4}$ in.	20° 25′ 30″
3 yd 1 ft 8 in.	$9\frac{1}{4}$ in.	6° 34′ 42″
8yd 5′4″	8′ $7\frac{1}{2}$″	30° 4′20″

$19\frac{1}{2}$
12
7

Subtract:

(d)	(e)
4 yd 2 ft 6 in.	40° 20′ 18″
2 yd 2 ft 8 in.	10° 28′ 30″
1 yd 2′ 10″	29° 51′ 48″

18 60

39
10
29

78
30
48

19
60
79
28
51

Work required on Questions 9 to 12.

9. Find the area in square inches of a surface 3 ft square.

10. Reduce 8.235 ft to feet, inches, and eighths of an inch.

11. An article whose weight is 1 lb $2\frac{1}{2}$ oz is packaged in a box that weighs $\frac{3}{4}$ oz. If 24 of these boxed articles are placed in a $2\frac{1}{2}$ lb shipping carton, what is the total weight, in pounds and ounces, of the shipping carton and its contents?

12. A plumber has a piece of pipe 20 ft long.
    a) What is the greatest number of pieces 2 ft $4\frac{3}{8}$ in. long that can be cut from it?
    b) Change 2 ft $4\frac{3}{8}$ in. to feet. Round off your answer to the nearest hundredth.

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